Electrogravity: On a scalar field of time and electromagnetism

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Abstract

It is possible to describe a universal scalar field of time but not a universal coordinate of time and to attribute its non-geodesic alignment to the electromagnetic phenomena. A very surprising outcome is that not only mass generates gravity, but also electric charge does. Charge is, however, coupled to a non-geodesic vector field and thus is not totally equivalent to inertial mass. Only the entire "Energy-Momentum" tensor has a vanishing divergence. The model can be seen as misalignment of physically accessible events in an observer spacetime and of gravity as a controlling response by volumetric contraction of the observer spacetime in the direction where events bend or accelerate to. This non geodesic acceleration is described by a generalization of the Reeb vector. Misalignment of events can be described by 1, 2, and 3 such vectors. The paper presents a term with 4 vectors but does not discuss its physical meaning. The paper also discusses particle mass ratios and the Fine Structure Constant where added or subtracted area in relation to a disk does not involve a ratio $\frac{1}{24}$ but $\frac{1}{96}$ due to the physical meaning of the orientation of a space foliation which is perpendicular to a time-like vector and due to the orientation of a plane which is perpendicular to a time-like vector and its Reeb vector. These two orientations mean that only one side of a 3-dimensional foliation has a physical meaning and only one side of a sub-plane of that foliation has a physical meaning then $\frac{1}{2}\frac{1}{2}\frac{1}{24} = \frac{1}{96}$. Another interpretation of the factor $\frac{1}{4}$ is the Bekenstein - Hawking entropy to area constant. An additional coefficient $\frac{4}{\pi}$ describes an acceleration field strength and has a compelling source in mainstream physics. Other two field strength coefficients are less understood but are very intuitive, these are $\frac{95}{96}$ and a critical value due to an imbalance equation between gravity and anti-gravity ~1.55619853719.

Keywords: General Relativity, Time, Electromagnetism.

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1. Introduction – measurement of non - geodesic deviation

The Result of the Geroch Splitting Theorem [1] is that a field of time can be defined. In simple geometries such as FRWL, which are Big Bang geometries, such time also has an intuitive meaning; it is a scalar field and not a coordinate of time. It is the maximal time between each event of space-time and the Big Bang as a limit, measured by a physical clock that may experience forces. Such proper time can be measured along different curves and is therefore not traceable, not geodesic under forces and cannot be a coordinate that also requires a 4-direction. The existence of a non – traceable time is not a new idea and was postulated by the philosopher R. Joseph Albo [2] in the 14th century. The approach that will be presented to make peace between General Relativity and Quantum Mechanics is not to describe Space-Time as emergent out of huge matrices and to preserve the particles approach [3], but to replace particles with events. In non-hyperbolic spacetime, a scalar field can still be defined as universal clock but will no longer be an upper limit of measurable time to an event from a Cauchy surface as an interpretation to [1].

What information can a scalar field encode, that is not already predicted by the metric tensor of space time $g_{\mu\nu}$? The answer is non - geodesic motion. The motion equations of the theory of General Relativity predict only geodesic motion. This theory is based on two assumptions,

1) The basic assumption is that matter is encoded via acceleration in the gradients of scalar fields, more specifically, the electromagnetic phenomena can be described by a non-zero acceleration of the gradient of a Geroch function [1] P^2 in hyperbolic space-time or PP* if P is complex. This acceleration is known as a Reeb vector field [4] in odd dimensions but can also be defined in 4 dimensions. Actions are defined for 1 Reeb field, "electromagnetic", 2 Reeb fields "electro-weak", 3 Reeb fields, "Strong" and 4 Reeb fields as a “Fifth Force”. A definition can be made also for 4 Reeb fields but its physical meaning is not discussed in this

paper. See appendix C, (65). The motivation to use Reeb vector fields, including a complex formalism, can be seen in the paper by Yaakov Friedman [5].

2) The scalar fields quantization is $P = \sum_{k=1}^{\infty} P(k)$ such that $\int_{\Omega} \frac{P(k)P^*(j)+P(j)P^*(k)}{2}\sqrt{-g}d\Omega = 0$ if $k \neq j$ and $\int_{\Omega} \frac{P(k)P^*(j)+P(j)P^*(k)}{2}\sqrt{-g}d\Omega = 1$ if $k = j$ where $\sqrt{-g}$ is the volume element of space-time, where g is the determinant of the metric tensor.

Note: The mathematical foundation of this paper is the Geroch function [1], [2], Reeb vector fields [4] for encoding trajectory curvature, symplectic geometry directly on spacetime and not on any phase space due to [5], and the idea of physically accessible events in an embedding spacetime, an idea very similar to Harland Snyder's quantized spacetime [6] but without any assumed non-commutative relation.

We can describe non geodesic integral curves along a field $P_\mu \equiv \frac{dP}{dx^\mu}$ for the coordinates x^μ, also, P_μ need not be time-like in all events of space-time. We now define the square norm for real numbers as $Z \equiv |P_\lambda P^\lambda|$ and its gradient $Z_\mu \equiv \frac{dZ}{dx^\mu}$. We define a geometric object $\frac{U_\mu}{2}$ that will measure how much the field P_μ is not geodesic.

$$U_\mu = \frac{Z_\mu}{Z} - \frac{Z_k P^k}{Z^2} P_\mu \Longrightarrow \tag{1}$$

$$\frac{d}{dx^\nu}\frac{P_\mu}{\sqrt{Z}} - \frac{d}{dx^\mu}\frac{P_\nu}{\sqrt{Z}} =$$

$$\frac{P_{\mu,\nu}}{\sqrt{Z}} - \frac{P_\mu Z_\nu}{2Z^{\frac{3}{2}}} - \frac{P_{\nu,\mu}}{\sqrt{Z}} + \frac{P_\nu Z_\mu}{2Z^{\frac{3}{2}}} =$$

$$\frac{P_\nu Z_\mu}{2Z^{\frac{3}{2}}} - \frac{P_\mu Z_\nu}{2Z^{\frac{3}{2}}} =$$

$$\frac{1}{2}\left(\frac{Z_\mu}{Z}\frac{P_\nu}{\sqrt{Z}} - \frac{Z_k P^k}{Z^2} P_\mu \frac{P_\nu}{\sqrt{Z}}\right) - \frac{1}{2}\left(\frac{Z_\nu}{Z}\frac{P_\mu}{\sqrt{Z}} - \frac{Z_k P^k}{Z^2} P_\nu \frac{P_\mu}{\sqrt{Z}}\right) = \frac{U_\mu}{2}\frac{P_\nu}{\sqrt{Z}} - \frac{U_\nu}{2}\frac{P_\mu}{\sqrt{Z}}$$

But why to use, $\frac{1}{2}U_\mu = \frac{1}{2}\left(\frac{Z_\mu}{Z} - \frac{Z_k P^k P_\mu}{Z^2}\right)$ and not simply, $\frac{Z_\mu}{Z}$? The reason is that $\frac{U_\mu}{2}\frac{P^\mu}{\sqrt{Z}} = 0$.

It is easy to show that $\frac{U_\mu}{2}$ behaves as the acceleration of the unit vector $\frac{P_\mu}{\sqrt{Z}}$. See Appendix D for another way to derive the Reeb vector. In terms of a 4-acceleration a_μ, it is easy to see:

$$\frac{U_\mu}{2} = \frac{a_\mu}{c^2} \tag{2}$$

Where c is the speed of light. $\frac{U_\mu}{2}$ is the generalization of a Reeb vector [4] to 4 dimensions. Can this a_μ have a simple physical meaning of accelerating any neutral mass ? There is an experimental way to find out, once we analyze the electric field in the coming sections.

To describe a field that accelerates any unit vector, we need an anti-symmetric matrix of acceleration similar to the Tzvi Scarr & Yaakov Friedman's acceleration matrix [7].

The matrix $A_{\mu\nu} = \frac{U_\mu}{2}\frac{P_\nu}{\sqrt{Z}} - \frac{U_\nu}{2}\frac{P_\mu}{\sqrt{Z}}$ is insufficient for that purpose; however, it can be extended quite easily, by using the Levi-Civita alternating tensor [8], not the alternating Levi-Civita symbol,

We have $B_{\mu\nu} = \frac{1}{2}E^{\mu\nu\alpha\beta}A_{\alpha\beta}$ which define an acceleration matrix in a perpendicular plane to the plane spanned by $\frac{P_\mu}{\sqrt{Z}}$ and $\frac{U_\mu}{2}$. In the complex case we define the acceleration matrix: $F_{\mu\nu} = A_{\mu\nu} + \gamma B_{\mu\nu}$ where $\gamma \epsilon U(1)$. With a vector w^ν, $w^\nu w_\nu = c^2$, we derive its acceleration,

$$F_{\mu\nu}\frac{w^\nu}{c} = \frac{a_{\mu(w)}}{c^2} \qquad (3)$$

$$\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{U_\mu U^\mu}{4}$$

Exercise to the reader: show that the Reeb vector $\frac{U_\mu}{2}$ of $\frac{P_\mu}{\sqrt{|Z|}}$ is the same as for $\frac{P_\mu}{\sqrt{|Z|}}e^{i\theta}$ for $i = \sqrt{-1}$ and a smooth scalar θ.

2. Electro-gravity

The action of gravity is defined as: $Action = Min \int_\Omega \left(R - \frac{1}{4ב}U^k U_k\right)\sqrt{-g}\, d\Omega$

The Euler Lagrange equations by the metric $g_{\mu\nu}$, by the scalar field of time P yield, Appendix A or [7]:

$$\frac{1}{4ב}\left(U_\mu U_\nu - \frac{1}{2}g_{\mu\nu}U_\lambda U^\lambda - 2U^k{}_{;k}\frac{P_\mu P_\nu}{Z}\right) = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} \qquad (4)$$

$$W^\mu{}_{;\mu} = \left(-4U^k{}_{;k}\frac{P^\mu}{Z} - 2\frac{Z_\nu P^\nu}{Z^2}U^\mu\right)_{;\mu} = 0$$

It is easy to prove without the right hand side that $\frac{1}{4ב}\left(U_\mu U_\nu - \frac{1}{2}g_{\mu\nu}U_\lambda U^\lambda - 2U^k{}_{;k}\frac{P_\mu P_\nu}{Z}\right)_{;}{}^{\nu}=0$ see Appendix B or [9]. (4) assumes $ב = 1$.

Theorem 1: If non-geodesic curves are prescribed to motion in material fields then zero Einstein tensor implies $\frac{1}{2}U_\mu = 0$, i.e. $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 0 \Longrightarrow \frac{1}{2}U_\mu = 0$ i.e. geodesic motion.

Proof:

We contract both sides of (4) with $U^\mu U^\nu$ so $\left(U_\mu U_\nu - \frac{1}{2}g_{\mu\nu}U_\lambda U^\lambda - 2U^k;_k \frac{P_\mu P_\nu}{Z}\right) U^\mu U^\nu = 0 \Longrightarrow$ $U_\lambda U^\lambda = 0$ because $U^\mu P_\mu = 0$ and now we contract both sides of (4) with $\frac{P^\mu P^\nu}{Z}$ so we have $\frac{P^\mu P^\nu}{Z}\left(U_\mu U_\nu - \frac{1}{2}g_{\mu\nu}U_\lambda U^\lambda - 2U^k;_k \frac{P_\mu P_\nu}{Z}\right) = -\frac{1}{2}U_\lambda U^\lambda - 2U^k;_k = 2U^k;_k = 0$ because $U_\lambda U^\lambda = 0$ and $\frac{P^\lambda P_\lambda}{Z} = 1$ so we get $U_\mu U_\nu - \frac{1}{2}g_{\mu\nu}U_\lambda U^\lambda - 2U^k;_k \frac{P_\mu P_\nu}{Z} = U_\mu U_\nu = 0 \Longrightarrow U_\mu = 0$. In other words, motion must be geodesic and we are done.

Remember $\frac{U_\mu}{2} = \frac{a_\mu}{c^2}$ as acceleration and the equation of gravity by Einstein, using the dust energy momentum tensor from General Relativity,

$$\frac{8\pi K}{c^4}T_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} \tag{5}$$

in (-,+,+,+) convention, we will use (5) further on, to show unique gravity by electric charge.

$$\frac{1}{4}U^k U_k = \frac{a^k a_k}{c^4} \tag{6}$$

(6) compared to Einstein's tensor means that the energy density in old physics terms can be seen as:

$$\frac{a^k a_k}{8\pi K \beth} = EnergyDensity \Longrightarrow \frac{8\pi K}{c^4}EnergyDensity = \frac{a^k a_k}{\beth c^4} = \frac{1}{4\beth}U^k U_k \tag{7}$$

Where $\beth = 1$ relates non geodesic acceleration to geometry, direct outcomes of (7) will be shown in (13) and (43). (7) means that the energy of the classical non-covariant electric field must be hidden in a very weak acceleration field

$$\frac{a^k a_k}{8\pi K \beth} \cong \frac{1}{2}\varepsilon_0 E^2 \tag{8}$$

ε_0 is the permittivity of vacuum, K is Newton's constant of gravity, which means

$$|a|^2 = 4\pi K \varepsilon_0 \beth E^2 \tag{9}$$

and

$$\|a^\mu\| = \sqrt{4\pi K \varepsilon_0 \beth}\|E\| \tag{10}$$

Indeed, a very weak acceleration if $\beth = 1$. However, there is a surprise:

$$\frac{1}{4ב}(U_\mu U_\nu - \frac{1}{2}g_{\mu\nu}U_\lambda U^\lambda - 2U^k{}_{;k}\frac{P_\mu P_\nu}{Z}) = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} \qquad (11)$$

Means that $\frac{1}{2ב}U^k{}_{;k} = \frac{a^k{}_{;k}}{c^2} = \sqrt{\frac{4\pi K\varepsilon_0 ב}{ב^2}}\frac{\rho}{\varepsilon_0 c^2} = \sqrt{\frac{4\pi K}{ב\varepsilon_0}}\frac{\rho}{c^2}$ where ρ is charge density.

Now remember the term $\frac{1}{4ב}(-2U^k{}_{;k}\frac{P_\mu P_\nu}{Z})$ and the relation $\frac{P^\mu P^\nu}{Z} \approx \frac{V^\mu V^\nu}{c^2}$ where $\frac{P^\mu}{\sqrt{Z}}$ is equivalent to a normalized velocity vector $\frac{V^\mu}{c}$, in Special Relativity $V^\mu = \frac{(c,v_x,v_y,v_z)}{\sqrt{1-v^2/c^2}}$, so we get

$$\frac{1}{8\pi K}\frac{U^\mu{}_{;\mu}}{2ב}\frac{P^\mu P^\nu}{Z^2} \approx \frac{1}{8\pi K}\sqrt{\frac{4\pi Kב}{ב^2\varepsilon_0}} \cdot \frac{\rho_{charge}V^\mu V^\nu}{c^4} = \frac{1}{8\pi K c^4}\sqrt{\frac{4\pi K}{ב\varepsilon_0}}\rho_{charge}V^\mu V^\nu \qquad (12)$$

But that can only mean that charge density behaves like mass density except for the fact that $\frac{P^\mu}{\sqrt{Z}}$ is not geodesic and therefore for charge Q:

$$M = \frac{Q}{\sqrt{16\pi K\varepsilon_0 ב}} \qquad (13)$$

Assuming $ב = 1$ where ε_0 is the permittivity of vacuum and K is Newton's constant of gravity, M is a gravitational mass, from (13) $\pm 1\ Coulombs$ is equivalent to $\pm\mathbf{5.802135215 * 10^9}$ **Kg**.

Caveat: $\frac{P^\mu}{\sqrt{Z}}$ is not geodesic unless $\frac{1}{2}U_\mu = 0$. So $\rho_{charge}\frac{P_\mu P_\nu}{Z}$ does not behave as inertial mass.

Electric field to acceleration from far observer coordinates – the following is not the way to derive the relation between gravitational mass and charge, not only because charge is coupled to a non-geodesic bivector, however, it does serve as an indication that the results are correct.

$$\frac{e}{4\pi\varepsilon_0 r^2}(4\pi\varepsilon_0 K)^{\frac{1}{2}} = \frac{c^2}{r} \qquad (13.1)$$

Where the right-hand side stands for acceleration or the norm of the Reeb vector multiplied by the squared speed of light. 'e' is the charge of the electron, ε_0 the permittivity of vacuum and K is the gravity constant of Newton. (13.1) is a result of (10).

$$\frac{e}{c^2}\left(\frac{K}{4\pi\varepsilon_0}\right)^{\frac{1}{2}} = r \qquad (13.2)$$

We will equate the right-hand side to the Schwarzschild radius of some mass,

$$\frac{e}{c^2}\left(\frac{K}{4\pi\varepsilon_0}\right)^{\frac{1}{2}} = \frac{2Km}{c^2} \qquad (13.3)$$

From which

$$e\left(\frac{1}{16\pi K\varepsilon_0}\right)^{\frac{1}{2}} = m \tag{13.4}$$

This is a very surprising result although it is not derived from the Euler Lagrange equations but just agrees with them 100% for the choice $ב = 1$ in (13).

Theorem 2: If the electromagnetic energy is not zero and the charge density $U^k;_k$ is zero in a domain D of space-time then U_0 is never 0 in all events of D.

Proof:

We write the Einstein - Grossmann equation (4) in its dual form, $R_{\mu\nu} = T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T_\alpha^\alpha =$
$\frac{1}{4ב}\left(U_\mu U_\nu - \frac{1}{2}g_{\mu\nu}U_\lambda U^\lambda - 2U^k;_k\frac{P_\mu P_\nu}{Z} - \frac{1}{2}g_{\mu\nu}g^{ij}\left(U_i U_j - \frac{1}{2}g_{ij}U_\lambda U^\lambda - 2U^k;_k\frac{P_i P_j}{Z}\right)\right) =$
$\frac{1}{4ב}\left(U_\mu U_\nu - \frac{1}{2}g_{\mu\nu}U_\lambda U^\lambda - 2U^k;_k\frac{P_\mu P_\nu}{Z} - \frac{1}{2}g_{\mu\nu}U^\lambda U_\lambda + g_{\mu\nu}U_\lambda U^\lambda + g_{\mu\nu}U^k;_k\right) = \frac{1}{4ב}(U_\mu U_\nu +$
$U^k;_k\,(g_{\mu\nu} - 2\frac{P_\mu P_\nu}{Z}))$. If $U_0 = 0$ in D then there exist local coordinates such that only the P_0 component of P_μ is not zero. We assumed $U^k;_k = 0$. Since $U_0 = 0$, $R_{00} = 0$ so the electromagnetic energy is zero. On the other hand, since U_μ is not zero, P_μ cannot be geodesic and therefore P_0 cannot be the only component of P_μ which is not zero along geodesic coordinates. Note: If there is a time-like curve γ around which U_μ is in relative motion in different events of every small D that contains γ, then R_{00} is not zero in D.

Note: There is one obvious peculiarity about charge generated gravity, $\frac{P^\mu}{\sqrt{Z}}$ is not the velocity of the charge. It is dictated by a scalar field of space-time!

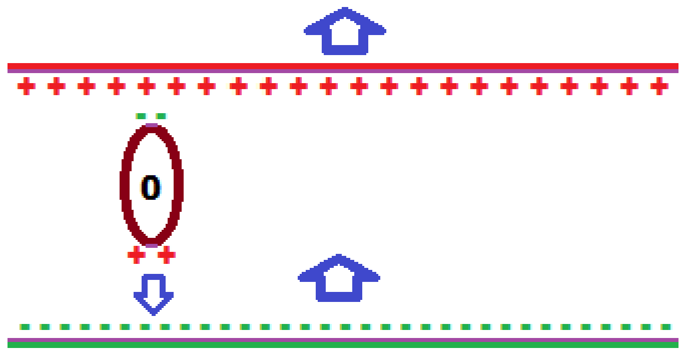
Note: From (10) and (13), if a^μ has a simple physical interpretation as a field that accelerates any neutral mass then we have to take (13) into account as an opposite effect. The result is that a field of 1,000,000 volts over 1 mm distance will accelerate any neutral particle at 8.61 cm * sec^{-2} and with taking into account (13) it will be less, due to an opposite gravitational effect, see (14), will be reduced to 4.305 cm * sec^{-2}.

The quantization of P is into a sum of event wave functions and has the physical meaning of Sam Vaknin's realization chronons [10]. The theory is easily expanded to 2 and to 3 Reeb vectors where the Lagrangian has U(1) SU(2) SU(3) symmetry if orientation is preserved, otherwise the symmetry group contains also reflections, see also an SU(4) Lagrangian, Appendix C. It is important to say that Vaknin's approach [10] is diametrically opposed to that of Jungjai Lee and Hyun Seok Yang [3].

3. Ceramic capacitors

In this section we will examine gravitational propulsion, not an Alcubierre's warp drive because the Alcubierre [11] extrinsic curvature condition $({K_i}^i)^2 - K_{ij}K^{ij} < 0$ will not hold in the same geometry as in the Alcubierre warp drive bubble. However, a negative plate below and a positive plate above, will manifest weak acceleration upwards as the negative gravity will push the positive plate upwards and the negative plate will be pulled by the positive plate above it. The main problem is that due to the dielectric material, the mass of the dielectric material will not be gravitationally repelled by the negative plate. Only a small portion of the mass of the capacitor will be affected in a highly dielectric material.

Fig. 1. – Only a small portion of the mass, in purple, is affected.



It is easy to see from (13) that in the classical limit near the plate, the gravitational field is mostly affected by charge density. By (13) the gravitational acceleration is

$$a \cong \frac{4\pi KQ}{A*\varepsilon*\sqrt{16\pi K\varepsilon_0 ℷ}} = \frac{V}{d\sqrt{ℷ}} * \sqrt{\pi K\varepsilon_0} \Longrightarrow \delta Weight \cong \frac{V}{d\sqrt{ℷ}} * \frac{M_{dielectric}}{g}\sqrt{\pi K\varepsilon_0} = \frac{V\rho A}{g\sqrt{ℷ}}\sqrt{\pi K\varepsilon_0} \quad (14)$$

where K is Newton's gravitational constant, Q is charge, A is area, ρ is the dielectric layer's density and M is its mass and ε_0 is the permittivity of vacuum, ε is the relative dielectric constant, assuming $ℷ = 1$, g is the Earth surface acceleration. (14) is the result of $Q = V\varepsilon_0\frac{A}{d} = \frac{V}{d}\varepsilon_0 A \Leftrightarrow \frac{Q}{\varepsilon_0 A} = 4\pi\frac{Q}{4\pi\varepsilon_0 A}E$ where E is the classical intensity of the electric field. We saw: $M = \frac{Q}{\sqrt{16\pi K\varepsilon_0 ℷ}}$ with $ℷ = 1$. The gravitational acceleration by the charge is $a \cong \frac{4\pi KM}{A} = \frac{4\pi KQ}{A*\varepsilon*\sqrt{16\pi K\varepsilon_0 ℷ}}$, if we assume an attenuation by the dielectric layer's induced dipoles to be proportional to the attenuation of the electric field by the same induced dipoles. This assumption is problematic because the induced dipoles are the accelerated material by the gravitational dipole of the external plates, and they are in much closer proximity to local charge than to the charge on the external plates. $\delta Weight \cong \frac{V}{d\sqrt{ℷ}} * \frac{M_{dielectric}}{g}\sqrt{\pi K\varepsilon_0} = \frac{V\rho A}{g\sqrt{ℷ}}\sqrt{\pi K\varepsilon_0}$ is therefore a very optimistic model.

Caution with (14): In reality, the charge of the induced dielectric dipoles is closer to the mass of the dipoles than the external plates. The assumptions of (14) therefore break down and the Inertial Dipole effect is much smaller. One possible technological remedy to this anti-alignment is to add an Alternating Current - AC component to the DC baseline and to disrupt the anti-alignment. Still, even with such a component, a feasible propulsion system may require millions of volts as a baseline. When using voltage above 2 * 511 kV, creation of electron-positron pairs is difficult to avoid, and the resulting gamma rays are a serious health hazard. A dynamic voltage and/or current component, renders the mathematical description of the Inertial Dipole much more difficult. The following calculations are therefore very optimistic.

Suppose we have a 1000Pf ceramic capacitor and we charge it with 10000 Volts and the area of the plates is 1 cm^2. The charge on the plates is then 10^{-5} Coulombs and its density 10^{-1} Coulombs per square meters. Now we want to calculate the approximate acceleration that the upper positive plate experiences due to the anti-gravity effect from the lower plate. Only a thin portion of the upper layer is affected, where the positive charge accumulates. A calculation shows: 0.48663510306 meters / sec^2. Dividing 0.4866351… meters/sec^2 by 9.81 meters / sec^2 we get 0.049606024776763 which is less than 5 percent relative to the gravity of the Earth. If instead of a dielectric material, an insulator with relative dielectric constant 1 is used for the same charge density of 10^{-5} Coulombs per 1 cm^2, a weight loss of the insulating slab should be measured at about 0.0496 of its weight. With a high relative dielectric constant, the affected mass could be well below 1 milligram, and it will lose 0.0496 of its weight. This renders the measurement of such an effect very hard to achieve unless the dielectric material is saturated and can no longer shield the field of the plates such as in the H4D experiment [12]. In any other case, practically no measurable thrust is expected for an area 1cm^2 with 10,000 Volts and scale resolution worse than 10^{-4} grams. In the case of saturation, at first the inertial dipole is expected to grow with the saturation of the dielectric material and with the amount of charge on the plates. [12] will be discussed later. The H4D lab [12] 69 mm radius and 2mm PMMA thickness capacitor with 20,000 volts, weight loss is at least **0.0015509 grams**, however the thickness of the metal plates is 1mm. It is sufficient to have a low frequency AC ripple from the DC power supply to churn the electrons on the plates such that not only a thin layer of the plates will be charged, also with an AC ripple, of typically 150 VAC for 20000 Volts DC, the induced gravitational field can no longer be considered static. Under such conditions (14) is no longer valid.

4. Thrust from 1000 Pf capacitor with two metallic plates and 10000 volts

Assumptions: Most of the dielectric mass is not completely shielded from the plate fields and the attenuation of the influence of the external dipole on the mass within the induced dipole is by a factor ε^{-1}, where ε is the relative dielectric constant. If this assumption does not hold true then (14) is invalid. Such a problem may occur at least theoretically even if in total the dielectric constant is low only because of low mass density. A second assumption is that dielectric dipoles are evenly distributed within the dielectric layer. A third assumption is a low alternating current – AC component in the power supply and that the influence of the Inertial Dipole on the metal plates is negligible due to the charge concentrating on the metallic surfaces which are in contact with the dielectric material. A high AC component might disrupt electrons alignment on the plates and if the plate's thickness is not negligible then (14) is no longer valid. Also, if the dielectric material reaches saturation and the metallic plates are thick in relation to the dielectric layer, the charge distribution on the plates can no longer be limited to the contact surfaces with the dielectric layers which also results in (14) being no longer valid.

Suppose we have a high voltage ceramic capacitor of 1000Pf of **Ta2O5** [13] with each plate area $1cm^2$ which is charged by 10,000 volts. The permittivity of vacuum is about 8.8541878128 * 10^{-12} Farads*meter^{-1}. So we can calculate the distance d between the plates, 8.8541878128 * 10^{-12} Farads * metere^{-1} * 10^{-4} meters^{2} * d^{-1} * 25 = 10^{-9} Farads. That means d ~ 0.22135469532 * 10^{-1} mm or d ~ 0.22135469532 * 10^{-2} cm. Now we take into account the weight density of the Ta2O5 which is 8.2 grams perm $1cm^3$ volume. So we have 8.2 * 1cm * 1cm * 0.22135469532 * 10^{-2} cm = 0.01815108501624 grams. At 10000 volts the weight loss is of a portion of 0.04960602477676315711411588216388 of the weight of the dielectric material and the inertial dipole is attenuated by the relative dielectric constant 25 just as the electric field is. So we have 0.01815108501624 grams * 0.04960602477676315711411588216388 * 25^{-1} ~ ~**3.60161*10^{-5} grams weight loss**. This estimate can be much lower in a multilayered capacitor where fields cancel out or when the dielectric constant is higher and the dipoles density is not uniform.

5. Martin Tajmar experimental null results analysis

Martin Tajmar [14] used a capacitor of a relative dielectric constant 4500 and a Teflon [15] capacitor with radius 50 mm and Teflon thickness d=1.5 mm and 10,000 Volts. The highly dielectric capacitor weight loss is way below the experiment **scale resolution 3 * 10^{-4} grams** due to division by 4500 of the charge which is 10^{-5} per 1000Pf capacitance. With a radius of 0.5cm, such a capacitor with say 6.02 grams * cm^{-3} density will lose about **2.077389 * 10^{-5} grams**. Next focus is on one of the Teflon capacitors. The gravitational acceleration on the face of the Earth, about g=9.80665 meter * sec^{-2}. By (14), the result is **7.5917876115 * 10^{-6} grams**. This result is smaller than the resolution of 3 * 10^{-4} grams. The results assume $ב = 1$ in (4), (7), (13). It is important to say that unlike Martin Tajmar (sounds as Taymar), the Brazilian H4D experiment [12] used much greater capacitor areas. A significant AC ripple cannot be ruled out.

6. Particle mass ratios

Motivation: solving (4) analytically is extremely hard, let alone, the more general Lagrangians that will be presented in (64) and (65) for complex Reeb vectors. One possible way to tackle this challenge is to rely on a theorem by Georges Reeb, according to which the restriction of the field to the three-dimensional foliation perpendicular to $\frac{P_\mu}{\sqrt{Z}}$, must have a zero rotor. In other words, the field must have drains and sources, by which the divergence of the field is not zero. The result of this theorem is that as the far observer $r \to 0$ in source or drain of the field, particles formation is inevitable. This section will try to find a relationship between an acceleration as $\sqrt{|a^\mu a_\mu|} = \xi' \frac{c^2}{r}$ for some ξ and the norm of the Reeb field $\sqrt{\frac{1}{8}\left|U^\mu U^*{}_\mu + U^{*\mu} U_\mu\right|} = \xi' \frac{1}{r}$ for some ξ'. In fact, this section considers (4) as $r \to 0$ as an attempt to avoid the extremely hard analytic solutions.

The following section will try to reach at the Reeb field strengths of the electron, Muon and Tau Lepton. It will also try to reach the Reeb field strength for the W and Z bosons. As we shall see, for the first 3 values, the assessment is $\frac{95}{96}$, $\frac{4}{\pi}$ and ~1.556198537190348396563877031439 9…

As we shall see $\frac{95}{96} = 1 - \frac{1}{64} + \frac{1}{192} = \frac{193}{192} + \frac{63}{64} - 1$, which can be interpreted as the summation of two fundamental states of the field. However, to keep an open mind, other possible reasons, although less plausible, are also brought into the discussion. The only value that does not come directly from this theory is $\frac{4}{\pi}$. It has a compelling Quantum Mechanics source; however, other less plausible explanations are also considered. The last value, ~1.55619853719, is derived from maximal imbalance between gravity and anti-gravity. For the W boson, two possible field strength coefficients are discussed $\frac{4}{\pi}$ and $\frac{4}{3}$. The latter yields a higher mass for the W boson although the author tends to accept $\frac{4}{\pi}$ and not $\frac{4}{3}$.

In this section, equation (4) is explored in a small infinitesimal sphere, where we assume a linear relation between a far observer radius r and acceleration $\frac{a^\mu}{c^2} = \frac{U^\mu}{2} = \frac{Z^\mu}{2Z} - \frac{Z^k P_k P^\mu}{2Z^2}$, see (1), (2). Our goal is to reduce (4) from a four-dimensional Minkowsky geometry to a three-dimensional Riemannian geometry and then to a two-dimensional Riemannian geometry of surfaces.

We make the following assumption:

$$\frac{\|a^\mu\|}{c^2} = \frac{\xi}{rx} \tag{15}$$

Where, c is the speed of light, ξ is a coefficient that depends on the field as $r \to 0$ and the variable x changes with the density of the field as it passes through a two-dimensional sphere. x is required because space-time curvature can cause such a sphere to be less than or more than $4\pi r^2$.

Note: A natural question due to (10) is, when does the acceleration $\frac{\xi c^2}{r} = \frac{e}{4\pi\varepsilon_0 r^2}\sqrt{4\pi\varepsilon_0 k}$, such that e is the charge of the electron, k is Newton's gravity constant, ε_0 is the permittivity of vacuum and c is the speed of light? The answer is $r = \frac{e}{\xi c^2}\sqrt{\frac{k}{4\pi\varepsilon_0}}$ and for $\xi = 1$, $r = \frac{e}{c^2}\sqrt{\frac{k}{4\pi\varepsilon_0}}$,which by the order of the inverse of the square root of the Fine Structure Constant is smaller than the Planck length, $\frac{e}{c^2}\sqrt{\frac{k}{4\pi\varepsilon_0}}\alpha^{-\frac{1}{2}} = \frac{e}{c^2}\sqrt{\frac{k}{4\pi\varepsilon_0}}\sqrt{\frac{4\pi\varepsilon_0\hbar c}{e^2}} = \sqrt{\frac{\hbar k}{c^3}}$, where $\hbar$ is the reduced Planck constant. This calculation of course, assumes that in such a strong field, the permittivity is that of vacuum and is not affected by virtual electric fields that attenuate the electric field. It is also limited to the far observer coordinates system.

We also make other assumptions as follows:

1) Assumption 1: In small radii, the energy of the gravitational field depends on the area around the source of gravity. This assumption is consistent with the paper of Ted Jacobson [16].

2) Assumption 2: The area ratio that has a physical meaning is between a disk to which the unit vector $\frac{P^\mu}{\sqrt{Z}}$ points to and the 1 weighted Euclidean sphere $ƛ * \pi r^2$ so $ƛ = 4$. The area loss of a disk is $\frac{\pi}{24}Rr^4$, where R is obtained by contracting Einstein's tensor twice with a time-like vector $\frac{P^\mu}{\sqrt{Z}}$ and r is an infinitesimal radius. However, we consider $\frac{1}{4}\frac{\pi}{24}Rr^4 = \frac{\pi}{96}Rr^4$. As we divide this area by Euclidean disk area, we get $\frac{\pi}{96}Rr^4 * (\pi r^2)^{-1} = \frac{1}{96}Rr^2$. Following are explanations to the factor $\frac{1}{4}$.

   Blackhole thermodynamics - Bekenstein and Hawking entropy and area: see the relation $S_{BH} = \frac{1}{4}K_B\frac{A}{{\ell_p}^2}$ [17] where A is area, ℓ_p is the Planck length, and K_B is Boltzmann's constant. We assume entropy is related to particles decay.

   Mathematically and physically compelling explanation: We return to the principles of the chronon field by Sam Vaknin [10] in which the time arrow is defined via spin and thus via orientation: There are two orientations to be considered. The first is the orientation of the foliation that is perpendicular to $\frac{P^\mu}{\sqrt{Z}}$. The second is the plane within that foliation which is perpendicular to $\frac{P^\mu}{\sqrt{Z}}$ and to $\frac{U^\mu}{2}$. In each case only one side of a 3D foliation and one side of a plane can be related to energy and $\frac{1}{2} * \frac{1}{2} = \frac{1}{4}$.

   Causal triangulation explanation: A polygonal graph is a graph in which vertices on a circle relate to edges and each vertex is also connected to the center. So, for the m vertices of the polygon and one vertex of the center, the graph has m+1 vertices. We also assume m=2n for some natural number n. The graph has 2m = 4n edges, m connecting the polygon vertices, each vertex to 2 neighbors and m connecting the polygon vertices to the center. Using graph theory techniques, it is easy to see that a random walk for a large n on such polygonal graph reaches a probability $\frac{1}{4}$ at the center and $\frac{3}{4m}$ at each polygon vertex. The probability of moving from a vertex on the polygon to one if its two neighbors is $\frac{1}{3}$ for each neighbor and to the center $\frac{1}{3}$. The probability of reaching one node of the polygon from the center is $\frac{1}{m}$. Seeing a particle as a loop with or without a center is beyond the scope of this paper, however, such a model under random walk reaches the unique probability $\frac{1}{4}$ at the center and is worth mentioning as another approach to area related to energy as $\frac{\pi}{96}Rr^4$ instead of $\frac{\pi}{24}Rr^4$, where R obtained by contracting Einstein's tensor twice with a timeline vector $\frac{P^\mu}{\sqrt{Z}}$ and r is an infinitesimal radius. The python code for the random walk calculations is brought here:

```
import numpy as NP
import numpy.linalg as LA

print('Random walk on 24-Polygonal  graph with a center.')
matrix = NP.zeros((25, 25), dtype=NP.float64)
a = 1/24
b = 1/3

for i in range(1, 25):
    matrix[0, i] = b
    matrix[i, 0] = a
    k = i + 1 if i < 24 else 1
    matrix[i, k] = b
    k = i - 1 if i > 1 else 24
    matrix[i, k] = b
    w, v = LA.eig(matrix)
    scale = v[:, 0].sum()
    v[:, 0] /= scale

print('Eigenvector of probability:')

for i in range(25):
    print(f'v[{i}]={v[i, 0]}')
    print(f'Eigenvalue {w[0]}')

The output is:
Random walk on 24-Polygonal graph with a center.
Eigenvector of probability:

v[0]=0.24999999999999997
v[1]=0.03124999999999989
...
```

The Causal Set interpretation (87)-(90) and its relation to the number 96 and the Fine Structure Constant cannot be ignored!

Non rigid explanation: This idea is derived from a physical principle according to which a spin of a particle always either points to an observer or in the opposite direction. In this manner, the observer can only refer to the disc which is perpendicular to the spin axis and not to an entire sphere. An area ratio $\frac{\pi r^2}{4\pi r^2} = \frac{1}{4}$ means 0 gravity.

This assumption means that the delta area of a curved sphere divided by $4\pi r^2$ is $\frac{\delta \pi r^2}{\lambda * \pi r^2}$ and not $\frac{\delta 4\pi r^2}{4\pi r^2}$. There could be other explanations to this assumption including a choice of $32\pi K$ in (7) instead of $8\pi K$ and $\frac{1}{16}$ instead of $\frac{1}{4}$ in (4), however to the author's opinion, (43) does not support such other explanations.

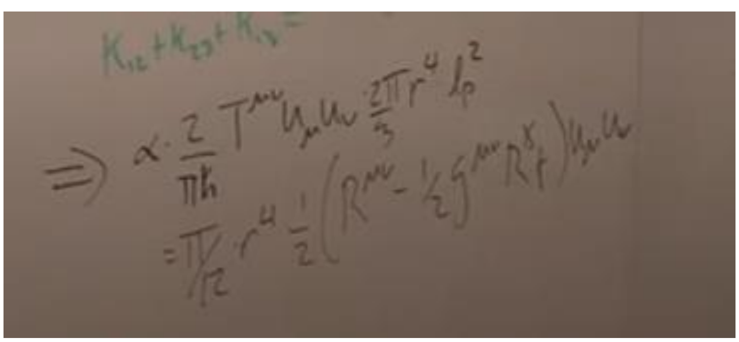
We revisit equation (4) and contract it twice with the unit vector $\frac{P^\mu}{\sqrt{Z}}$ which means a chosen time direction $\frac{1}{4\beth}\left(U_\mu U_\nu - \frac{1}{2} g_{\mu\nu} U_\lambda U^\lambda - 2U^k{;_k} \frac{P_\mu P_\nu}{Z}\right) \frac{P^\mu P^\nu}{Z} = (R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}) \frac{P_\mu P_\nu}{Z}$

Since $U_\mu P^\mu = 0$, and assuming $\beth = 1$, we have around an electric charge by (15)

$$\frac{1}{\beth}(-\frac{1}{2} g_{\mu\nu} \frac{U_\lambda U^\lambda}{4} - \frac{1}{2} U^k{;_k}) = \frac{1}{\beth}(-\frac{1}{2}\frac{\xi^2}{r^2 x^2} \mp \frac{\xi}{r^2 x}) = (R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}) \frac{P_\mu P_\nu}{Z} \qquad (16)$$

We calculated the divergence of a field of a non-geodesic acceleration from intensity $\frac{\xi}{rx}$ to 0 along the distance r. The divergence $U^k{;_k}$ can be either positive or negative and depends on the sign of the electric charge. We now refer to Seth Lloyd lecture [18],

Fig. 2. Area gain or loss in the direction of a unit vector:



As we see, to get the area loss on a disk which is perpendicular to the unit vector $\frac{P^\mu}{\sqrt{Z}}$ due to curvature, we need to multiply (16) by $\frac{\pi}{12}\frac{1}{2} r^4 = \frac{\pi}{24} r^4$.

$$\frac{1}{\beth}\left(-\frac{1}{2}\frac{\xi^2}{r^2 x^2} \mp \frac{\xi}{r^2 x}\right)\frac{\pi}{24} r^4 = \frac{1}{\beth}\left(-\frac{1}{2}\frac{\xi^2}{x^2} \mp \frac{\xi}{x}\right)\frac{\pi}{24} r^2 = AreaLossOfADisk \qquad (17)$$

By our second assumption, the following has a physical meaning, where $ג = 4,\ ב * ג = 1 * 4 = 4$

$$\left(-\frac{1}{2}\frac{\xi^2}{x^2} \mp \frac{\xi}{x}\right)\frac{1}{96} = \frac{1}{ב*ג}\frac{1}{\pi r^2}\left(-\frac{1}{2}\frac{\xi^2}{x^2} \mp \frac{\xi}{x}\right)\frac{\pi}{24}r^2 = \frac{AreaLossOfADisk}{ג*\pi r^2} \tag{18}$$

But x should be a ratio between an area around a charge and Euclidean area, according to assumption 2. If x is greater than 1, then by (17), the non-geodesic acceleration field density is decreased by a factor of $\frac{1}{x}$. If the area ratio is smaller 1 then the non geodesic field density is increased by $\frac{1}{x}$. So we must have the following equation:

$$x = 1 + \frac{AreaLossOfADisk}{4\pi r^2} \Leftrightarrow x - 1 = \frac{AreaLossOfADisk}{4\pi r^2}$$

And by (10) and (12), (18) becomes:

$$\left(-\frac{1}{2}\frac{\xi^2}{x^2} \mp \frac{\xi}{x}\right)\frac{1}{96} = x - 1 \Leftrightarrow 1 + \left(-\frac{1}{2}\frac{\xi^2}{x^2} \mp \frac{\xi}{x}\right)\frac{1}{96} = x \Leftrightarrow \frac{192x^2 \mp 2\xi x - \xi^2}{192} = x^3 \tag{19}$$

The righthand side is expected to be positive around a negative charge and negative around a positive charge if we take into account the H4D experimental qualitative result [12] with imprecise balance.

We will first start with an assumption $\xi = \frac{4}{\pi}$. This assumption is based on Ettore Majorana's notebook [19] and on the compelling assessment of the critical strength of the Coulomb and the Yukawa potentials [20]. It is also the well-known ratio between a star graph and a Steiner star in Euclidean spaces – *star Steiner ratio* in $\mathbb{R}^d$ [21]. The addition of a middle point in a ball can reduce the length of a star graph in relation to a star where the star graph is defined as straight lines between n-1 points and a single point on the sphere. And a Steiner star connects the points to the center. A physical meaning of such a ratio is that where there is a middle point, divergence of an acceleration field can be defined, where there is no such point, no such divergence can be defined. For such a case, a different value of ξ should be defined.

Then (19) yields two solutions as follows,

$$\frac{192{x_1}^2 + 2\xi x_1 - \xi^2}{192} = {x_1}^3 \Longrightarrow \frac{1}{x_1 - 1} \cong \mathbf{206.75133988502202} \tag{20}$$

This value is surprisingly very close to the mass ratio between the Muon and the electron!

$$\frac{105.6583745\text{MeV}}{0.5109989461\text{MeV}} \cong 206.7682826 \tag{21}$$

The following is an area ratio around a positive charge. The discussion about its meaning is postponed for now.

$$\frac{192{x_2}^2-2\xi x_2-\xi^2}{192} = {x_2}^3 \Longrightarrow \frac{1}{1-x_2} \cong \mathbf{44.63955017596401} \tag{22}$$

Before we continue, we need to prove another theorem which has important implications to Quantum Gravity. The factor $\frac{96}{95}$ is, however, not final in what will be described as Steiner Trees.

Theorem 3: In Riemannian geometry, a computational model for the connection of a finite connected set of points on a sphere S2 and the center with radius r can converge in polynomial time only to a minimal graph of S2 not within radius r but within radius $r\frac{96}{95}$.

Proof: The proof of this theorem is a direct result of the complexity limit of the Minimum Steiner Tree. Finding the minimal length of such a graph is in polynomial time only above $\frac{96}{95}$ of the minimal graph length due to [22]. As a result, to connect all the points in the sphere and its center is possible in polynomial time only for $r\frac{96}{95}$ and we are done. The meaning of this theorem is very deep for most Quantum Gravity theories. For this specific theory, if acceleration depends on r^{-1} then physically the dependence must be on $\frac{95}{96}r^{-1}$. As a caveat, $\frac{96}{95}$ is not believed by the author to be an absolute limit to the hardness of the Steiner Tree problem.

Consider $\xi = x_1$ and $\xi = x_2$ in the following area ratio equations.

$$\xi_1 = x_1 \ \wedge \ \left(\frac{1}{2}\frac{{\xi_1}^2}{{x_1}^2} + \frac{\xi_1}{x_1}\right)\frac{1}{96} = x_1 - 1 \Longrightarrow x_1 = \frac{193}{192} \Leftrightarrow \ \delta x_1 = x_1 - 1 = \frac{1}{192} \tag{22.1}$$

$$\xi_2 = x_2 \ \wedge \ \left(\frac{1}{2}\frac{{\xi_2}^2}{{x_2}^2} - \frac{\xi_2}{x_2}\right)\frac{1}{96} = x_2 - 1 \Longrightarrow x_2 = \frac{63}{64} \Leftrightarrow \ \delta x_2 = x_2 - 1 = \frac{-1}{64} \tag{22.2}$$

Adding these two delta area ratios yields

$$\delta x_1 + \delta x_2 = \frac{1}{192} + \frac{-1}{64} = -\frac{1}{96} \tag{22.3}$$

$$\xi = 1 + \delta x_1 + \delta x_2 = \frac{95}{96} \tag{22.4}$$

Definition: $\xi_1 = \frac{193}{192}$ and $\xi_2 = \frac{63}{64}$ will be called Stability Field Strengths and $\xi = \frac{95}{96}$ is called Joint Stability Field Strength. $\xi = \frac{95}{96}$ is the first candidate for the electron field strength that will be used in the Muon/electron mass ratio assessment. It is not difficult to see that for the choice of $\xi = \frac{95}{96}$, also see motivation in Appendix E, (74), (75), (79), and the surprising relation between the Fine Structure Constant and exponential perturbations of $\xi = \frac{95}{96}$ in (81)-(86), the following polynomials yield,

$$\left(-\frac{1}{2}\frac{(\frac{95}{96})^2}{a^2}+\frac{\frac{95}{96}}{a}\right)\frac{1}{96}=a-1\Longrightarrow\frac{192a^2+2\frac{95}{96}a-(\frac{95}{96})^2}{192}=a^3\ and\ \left(-\frac{1}{2}\frac{\left(\frac{95}{96}\right)^2}{b^2}-\frac{\frac{95}{96}}{b}\right)\frac{1}{96}=b-1\Longrightarrow$$

$$\frac{192b^2-2\frac{95}{96}b-(\frac{95}{96})^2}{192}=b^3\ and\ \frac{1}{(a-1)(1-b)}\cong\mathbf{12202.88874066467724}\qquad(23)$$

$(a-1)(1-b)$ answers the question of what happens when the test particle is neutral. To better understand the above expression, it is best to contract the acceleration matrix $A_{\alpha\beta}$ (3), [7] with the Levi-Civita tensor (not symbol), $E^{\mu\nu\alpha\beta}$ but with a possible orientation change from $B_{\mu\nu}=\frac{1}{2}E^{\mu\nu\alpha\beta}A_{\alpha\beta}$. This description is of a second plane in which the divergence of a Reeb-like acceleration vector can be of an opposite sign, $\frac{\tilde{U}^{\mu}}{2}=\frac{1}{2}E^{\mu\nu\alpha\beta}A_{\alpha\beta}V_{\upsilon}$ where V_{υ} is a unit vector perpendicular to both $\frac{U^{\mu}}{2}$ and to $\frac{P^{\mu}}{\sqrt{Z}}$. One field is then of a positive charge and one of a negative charge which is the explanation for the term $(a-1)(1-b)$ where a denotes the area addition ratio around a negative charge and b is the area loss ratio around a positive charge. One would expect to see $\sqrt{(a-1)(1-b)}$ however, roots will be discussed regarding spin 1 mass ratios.

Roots of such a value also have a meaning, see appendix C, (64). Combining (20) and (23), the following holds:

$$\frac{(x_1-1)\mathbf{105.65837455}MeV}{1+(a-1)(1-b)}\cong\mathbf{0.5109989461}MeV$$

$$1+\frac{1}{96}\left(-\frac{1}{2}(1-\frac{1}{96})^2a^{-2}+\left(1-\frac{1}{96}\right)a^{-1}\right)=a$$
$$1+\frac{1}{96}\left(-\frac{1}{2}(1-\frac{1}{96})^2b^{-2}-\left(1-\frac{1}{96}\right)b^{-1}\right)=b$$
$$1+\frac{1}{96}\left(-\frac{1}{2}\left(\frac{4}{\pi}\right)^2c^{-2}+\frac{4}{\pi}c^{-1}\right)=c$$
$$MuonMass*(c-1)=ElectronMass+ElectronMass*(a-1)(1-b)\qquad(24)$$

By (23) the ratio is **~206.76828270441461654627346433699131011962890625**

Where $ElectronMass*(a-1)(1-b)=\sim\mathbf{41.875}\ \boldsymbol{eV}/c^2$ looks like a new particle or resonance. Corroboration requires to detect excess in cosmic ~20.9 $\boldsymbol{e}V/c^2$ photons. Verification of this theory by Muon decays can be done by observing rare excess of **20.9376221059304 eV photons**. With electron energy **0.5109989500 MeV** the Muon energy is **~105.658375355 MeV**.

We only needed a small correction to the 2014 Muon energy from **105.6583745 MeV** to **105.65837455 MeV** with electron energy **0.5109989461055 MeV** to arrive at the energy ratio and therefore mass ratio of the Muon and the electron. Is that a mere coincidence? The extremely small ratio error and the choices of $\xi=\frac{4}{\pi}$ and $\xi=\frac{95}{96}$ highly disfavors a mere coincidence. It is important to notice that $1+\frac{1}{96}\left(-\frac{1}{2}(\frac{193}{192})^2a^{-2}+\left(\frac{193}{192}\right)a^{-1}\right)=a$ has a biggest root $a=\frac{193}{192}=1+\frac{1}{192}$ and $1+\frac{1}{96}\left(-\frac{1}{2}(\frac{63}{64})^2b^{-2}-\left(\frac{63}{64}\right)b^{-1}\right)=b$ has a biggest root $b=\frac{63}{64}=1-\frac{1}{64}$. The delta $-\frac{1}{64}+\frac{1}{192}=-\frac{1}{96}$ is a delta of energy ratios between the two stable states with field strength coefficients $\xi=\frac{193}{192}$ and $\xi=\frac{63}{64}$ and roots $a=\frac{193}{192}$ and $b=\frac{63}{64}$. 1 plus this delta yields $\frac{95}{96}$, which shows that our choice of $\xi=\frac{95}{96}$ was not at random but is the result of the summation of negative and positive area ratios for which the field strengths are equal to the biggest roots.

Let us define the electron field strength as $\xi=\frac{95}{96}$ and consider a perturbation of this value and its close link to the Fine Structure Constant. Recall (23),

$$1+\frac{1}{96}\left(-\frac{1}{2}(1-\frac{1}{96})^2a^{-2}+\left(1-\frac{1}{96}\right)a^{-1}\right)=a \tag{24.1}$$

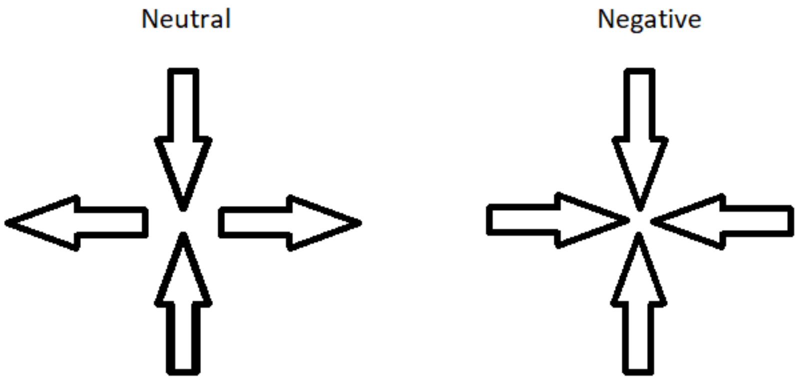
$$1+\frac{1}{96}\left(-\frac{1}{2}(1-\frac{1}{96})^2b^{-2}-\left(1-\frac{1}{96}\right)b^{-1}\right)=b$$

Consider a little more accurate result than the one used before in (23):

$$\frac{1}{(a-1)}\frac{1}{(1-b)}\cong 12202.888740664679062319 9 \tag{24.2}$$

Now consider the following perturbation on $\xi=\frac{95}{96}=1-\frac{1}{96}$, and raise this $\frac{95}{96}$ to the power $1+\alpha$ where α is the Fine Structure Constant - FSC. We will take the assessment from (40) of the inverse FSC, about 137.035999036827007557 8… and not the larger assessment 137.0359992990990 from the remark after (41). Our new field strength will be $\xi'=(\frac{95}{96})^{1+\alpha}$, which is an exponential perturbation with the help of the Fine Structure Constant. We want to calculate the new value of the neutral area ratio of addition and subtraction in two acceleration planes, one positive and one negative, as expected from the electron Neutrino because it is electrically neutral and see what we get. Before that, please view the following illustration, in reality, it is a 4-dimensional model with two perpendicular planes. The neutral charge is of one positive two-dimensional plane and one negative two-dimensional plane, both defined by two acceleration matrices and two generalizations of two Reeb vectors to 4 dimensions.

Fig. 3.: this is an over-simplification of two Reeb fields in two Symplectic Lagrangian planes:



And now we have for $\xi' = (\frac{95}{96})^{1+\alpha}$,

$$\frac{1}{(a\prime - 1)} \frac{1}{(1 - b\prime)} \cong 12204.188931677483196836 \tag{24.2}$$

And the following shows up:

$$(1 - \frac{(a' - 1)(1 - b')}{(a - 1)(1 - b)})^{-\frac{1}{2}} \cong 96.8837368186132295022617 \tag{24.3}$$

which is remarkably close to $\alpha^{-1} 2^{-\frac{1}{2}} \cong 96.8990841856135745047140052$, from (40).

The fact that perturbations of the field strength $\frac{95}{96}$ yield the Fine Structure Constant is easy to see in other cases other than (81)-(86) and (24.3). Using a simple datasheet without the accuracy of the Python math libraries, consider $\xi' = \left(\frac{95}{96}\right)^{1+\beta}$ where $\beta \approx 1.00370694$ for which $(\frac{95}{96} - \xi')(\frac{95}{96})^{-1} \cong 25762.75334^{-1}$ then (23) yields $\frac{1}{(a\prime - 1)} \frac{1}{(1 - b\prime)} \cong 12203.54919$. Now consider the following function $\xi'' = 1 + \ln(\xi')$ for which (23) yields $\frac{1}{(a\prime\prime - 1)} \frac{1}{(1 - b\prime\prime)} \cong 12204.49567$,

$$\ln\left(\frac{\ln((a'' - 1)(1 - b''))}{\ln((a' - 1)(1 - b'))} - 1\right)^2 \approx 137.0359991 \tag{24.4}$$

In other cases, the inverse Fine Structure Constant can emerge from trigonometric perturbations of a higher field strength. In both cases, fractional powers of roots are involved. This is not surprising if we consider that the Fine Structure Constant must be related to electromagnetic waves, and these should be a result of perturbations of the field strength of elementary particles such as the electron or even of the Tau lepton as an upper limit of an allowed leptonic field strength. Although (24) is not a rigid mathematical proof of the mass ratios between the Muon and the electron, and although only $\xi = \frac{4}{\pi}$ is a well understood field strength, not directly from this paper, one can argue that the result in (24) is too accurate to be ignored, especially if (24.3), (24.4) and (81)-(86) in "Appendix E" are taken into account.

$\xi = 2$ as a field strength is a critical value much higher than the highest field strength for leptons which is offered in this paper, simply because for a negative charge, the gravitational field vanishes.

$\frac{192x^2+2*2*x+2^2}{192} = x^3$ with a stable root x=1, $x(n+1) = \left(\frac{192x(n)^2+2*2*x(n)-2^2}{192}\right)^{\frac{1}{3}}$. But then $\frac{1}{x-1}$ as an added area portion around the negative charge is undefined but with a left limit 0. So, asking whether a logarithmic scale that starts at 2 has a physical meaning is legitimate. We choose our scale to be:

$\{2^{\frac{95*96}{95*96}}, 2^{\frac{95*96-1}{95*96}}, \ldots, 2^{\frac{1}{95*96}}\}$, now consider $y = \left(\left(2^{\frac{1}{95*96}} - 1\right) * 96\right)^{-1}$ ≅137.050820617 and

$\frac{95}{\ln(2)} \cong 137.05602888445$ and it is easy to show that as n grows, $\left(\left(2^{\frac{1}{(n-1)*n}} - 1\right) * n\right)^{-1} \approx \frac{n-1}{\ln(2)}$. It is easy to see a nice result, $\frac{y-137.0359990368270075578}{137.0359990368270075578} \cong 96.1546032^{-2}$ so the relative error to one of the assessment of the inverse Fine Structure Constant, see (40), is nearly expressible as a power of 96. This is one good reason to search for a relation that involves 2 and powers of 96 or of 95*96 as the mathematical term that will yield the Fine Structure Constant, however, such a term should appear out of a perturbation of a field strength because the Fine Structure Constant defines the Quantum electric strength, but which field strength?

Important: A leading idea is that the Fine Structure Constant should be related to perturbations of a maximal allowed field strength for leptons, i.e., the Tau lepton field strength. Any perturbation exceeding this limit must be dissipated as waves.

The exploration which is performed here is not out of analytic solutions to (4) or a complex version of (4) or to the further-on mentioned (64), which may take many years to yield fully analytic solutions. It is a "reverse engineering" of Nature by assessment of (4) and field strengths in an infinitesimal limit. It will require more discussion to reach more comprehensible terms for the inverse Fine Structure Constant.

Another clue to where the Fine Structure constant comes from is the following:

Consider a search for the number 96 and to keep the idea simple and related to the roots of the third order Gravity and Anti-gravity area ratio polynomials.

Consider the following known equation: $\frac{\pi^4}{96} = \sum_{k=0}^{\infty} \frac{1}{(2k+1)^4}$ which results from Parseval's identity when developing the Fourier series of the function $f(x) = |x|$ in $(-\pi, \pi)$. Notice that the fourth root of $\frac{\pi^4}{96}$ is $\frac{\pi}{96^{\frac{1}{4}}} \cong 1.00364948118 \ldots$. We can see what the error of this value in relation to 1 is. $\frac{\pi}{96^{\frac{1}{4}}} - 1 \cong \frac{1}{274.01155134419542\ldots} = \frac{1}{2*137.00577567209771179617192026613}$. We may therefore search for an expression in which twice the inverse Fine Structure Constant appears. If we choose the value from (40), and not the higher value 137.0359992990990 after (41) we get the following error assessment

$$\left(1 - \frac{137.00577567209771179617192026613}{137.03599903682700755780388135 46}\right) \cong \frac{2}{(95.22718072640602888004043636 2512)^2}$$

The residual error is related to a number between 95 and 96 and the factor 2 appears again. Although it is not any mathematical proof, it is still difficult to ignore such a lead in the search for where the inverse Fine Structure comes from.

It is worthy of mentioning that getting the exact Fine Structure Constant in (81)-(86) requires a very small addition $\xi = \frac{95}{96} + \varepsilon$ for some small ε. Then (24) would require the mass of the Muon to be slightly higher than 105.65837455 MeV, about 105.658375 MeV.

The following Python code was used to reach the result in (24),

```
import numpy as np

x1 = 1
third = 1 / 3
f = 4 / np.pi # Ettore Majorana's ring of a disk, potential factor.
f2 = f * f

# Iterate to most stable root.
for i in range(2000):
    x1 = np.power((192 * x1 * x1 + 2 * x1 * f - f2) / 192, third)

a = 1/(x1 - 1) # Negative charge.

print('Xi = 4/Pi, a = %.48f' % a)

x3 = 1
x4 = 1
f = 95 / 96
f2 = f * f

# Iterate to most stable roots.
for i in range(2000):
    x3 = np.power((192 * x3 * x3 + 2 * x3 * f - f2) / 192, third)
    x4 = np.power((192 * x4 * x4 - 2 * x4 * f - f2) / 192, third)

c = 1/(x3 - 1) # Negative charge.
d = 1/(1 - x4) # Positive charge.
```

```
print('Xi = 95/96, c = %.48f, d = %.48f' % (c, d))
print('Xi = 95/96, c * d = %.48f' % (c * d))

print('Approximated mass ratio between the Muon and the electron %.48f'
      % (a * (1 + (x3-1)*(1-x4))))
```

Few words about $\xi = 1 - \frac{1}{96}$. What is so special about $\xi = 1 - \frac{1}{96}$? It is twice the average of an ideal area loss ratio and an ideal area addition ratio + 1. $\frac{1}{192} - \frac{1}{64} = -\frac{1}{96}$ where $x = 1 + \frac{1}{192}$ is the biggest root of $1 + \frac{1}{96}\left(-\frac{1}{2}(1 + \frac{1}{192})^2 x^{-2} - \left(1 + \frac{1}{192}\right) x^{-1}\right) = x$ and $x = 1 - \frac{1}{64}$ is the biggest root of $1 + \frac{1}{96}\left(-\frac{1}{2}(1 - \frac{1}{64})^2 x^{-2} - \left(1 - \frac{1}{64}\right) x^{-1}\right)$.

How about null Reeb vectors $\frac{U_\mu}{2}\frac{U^\mu}{2} = 0$. It is not difficult to see that in this case, the unit vector $\frac{P_\mu}{\sqrt{Z}}$ should be space-like at least in the near vicinity of the test particle as $r \to 0$ and U^μ may not be all 0 at the center of a sphere but can be a null vector. With $\xi = \frac{4}{\pi}$ and $\xi = \frac{95}{96}$ we have in this case:

$$1 + \frac{1}{96}\left(\pm\frac{4}{\pi} c^{-1}\right) = 1 \pm \frac{c^{-1}}{24\pi} = c \qquad (25)$$

and

$$1 + \frac{1}{96}\left(\pm\frac{95}{96} b^{-1}\right) = 1 \pm \frac{95 b^{-1}}{96^2} = b \qquad (26)$$

From (25)

$$c_1 = \frac{1 + \left(1 + \frac{1}{6\pi}\right)^{\frac{1}{2}}}{2} \cong 1.0130915 \ldots, \; c_2 = \frac{1 + \left(1 - \frac{1}{6\pi}\right)^{\frac{1}{2}}}{2} \cong 0.986556 \ldots \text{ and} \qquad (27)$$

From (26)

$$b_1 = \frac{1 + \left(1 + \frac{95}{96*24}\right)^{\frac{1}{2}}}{2} \cong 1.010204037 \ldots, \; b_2 = \frac{1 + \left(1 - \frac{95}{96*24}\right)^{\frac{1}{2}}}{2} = \frac{95}{96} \;\text{ and}$$

$$\frac{1}{c_1 - 1} \cong 76.38530, \; \frac{1}{1 - c_2} \cong 74.3845968, \; \frac{1}{\sqrt{(c_1 - 1)(1 - c_2)}} \cong 75.3783115, \qquad (28)$$

With $\xi = \frac{4}{3}$, (28) is a bit different:

$$\frac{1}{c_1 - 1} \cong 72.98648402, \; \frac{1}{1 - c_2} \cong 70.98571137, \; \frac{1}{\sqrt{(c_1 - 1)(1 - c_2)}} \cong 71.9791462, \qquad (28.1)$$

Important: Where does this $\xi = \frac{4}{3}$ come from? The reader is advised to check that the average distance between two points on the Euclidean ring is $\xi = \frac{4}{\pi}$. The average distance between two points on the Euclidean sphere is $\xi = \frac{4}{3}$ and is left as an exercise to the reader. We may say that

$\xi = \frac{4}{\pi}$ means a geometric ring field strength and $\xi = \frac{4}{3}$ is a geometric sphere field strength. If we take into account particle decay through Bosons with two different field strengths, $\xi = \frac{4}{\pi}$ if the Muon is involved and $\xi = \frac{4}{3}$ in other cases, then there is a new interaction that is not covered by the W Boson alone!

$$\frac{1}{b_1-1} \cong 98.00042535, \ \frac{1}{1-b_2} = 96, \ \frac{1}{\sqrt{(b_1-1)(1-b_2)}} \cong 96.99505572 \tag{29}$$

We now look at:

$$\frac{\sqrt{(b_1-1)(1-b_2)}}{\sqrt{(c_1-1)(1-c_2)}} \cong 1.134361808^{-2} \tag{30}$$

Roots are attributed in this case to spin 1 or 2. It is easy to see that also:

$$(1+(c_1-1)(1-c_2))\left(\frac{(c_1-1)(1-c_2)}{(b_1-1)(1-b_2)}\right)^{1/4} \cong 1.134561453 \tag{31}$$

$$\approx \frac{91.1876\ GeV}{80.3725\ GeV}$$

Which is remarkably close to the ratio between the energy of the Z boson and the energy of the W boson and for W Boson of 80.3725 GeV the relative error of this ratio is about 1/1528961.689. For where the idea of 4$^{\text{th}}$ roots came from, please refer to Appendix C, (65).

Another research direction is to use the inverted value of $\xi = \frac{4}{\pi}$, i.e., $\xi = \frac{\pi}{4}$ in the negative and positive charge area ratio equations as in (24). That yields two new maximal roots $a_1{}^2 + \frac{1}{96}\left(-\frac{1}{2}\left(\frac{\pi}{4}\right)^2 + \frac{\pi}{4}a_1\right) = a_1{}^3$ and $a_2{}^2 + \frac{1}{96}\left(-\frac{1}{2}\left(\frac{\pi}{4}\right)^2 - \frac{\pi}{4}a_2\right) = a_2{}^3$ along with the older ones $b_1{}^2 + \frac{1}{96}\left(-\frac{1}{2}\left(\frac{4}{\pi}\right)^2 + \frac{4}{\pi}b_1\right) = b_1{}^3$ and $b_2{}^2 + \frac{1}{96}\left(-\frac{1}{2}\left(\frac{4}{\pi}\right)^2 - \frac{4}{\pi}b_2\right) = b_2{}^3$. Quite like the ratio in (30), we have, $\frac{\sqrt{(b_1-1)(1-b_2)}}{\sqrt{(a_1-1)(1-a_2)}} \cong \sqrt{\frac{201.6240447 * 86.46523917}{206.7513399 * 44.63955018}}$ ≅1.374383282 which is close to the following mass ratio between a Higgs Boson of 125.3267 GeV and a Z Boson of 91.1876 GeV which yields, 1.37438314, close to 1.374383282. It is interesting though not sufficiently accurate to draw any conclusion at this stage. The idea behind using charge equations without null Reeb vectors is because the Higgs boson is supposedly responsible for non-zero mass. From (31) and using s instead of c, $\sqrt{(s_1-1)(1-s_2)} \cong 75.3783115\ ...^{-1}$ and 91.1876 GeV * $\frac{\sqrt{(b_1-1)(1-b_2)}}{\sqrt{(a_1-1)(1-a_2)}}$ * $(1+(s_1-1)(1-s_2)) \cong$ 125.3487702 GeV. A similar $(1+(s_1-1)(1-s_2))$ value was used in (29) as $(1+(c_1-1)(1-c_2))$. If the reasoning here is correct, the Higgs boson interacts as an electric dipole.

Returning to (22) $\frac{1}{1-x_2} \cong \mathbf{44.63955017596401}$ and written as $\frac{1}{1-c}$,

$$\frac{80372.88 \text{ MeV } (1-c)}{1+\sqrt{(c_1-1)(1-c_2)}} \approx \mathbf{1776.91\ MeV} \tag{32}$$

With $\xi = \frac{4}{3}$ as in (28.1), (32) gets the same result for a higher value of the W Boson mass,

$$\frac{80422.57 \text{ MeV } (1-c)}{1+\sqrt{(c_1-1)(1-c_2)}} \approx \mathbf{1776.91\ MeV} \tag{32.1}$$

The root, $\sqrt{(c_1-1)(1-c_2)}$ can be better understood as a result of taking the root of a determinant of a Gram matrix of two Reeb vectors in Appendix C or is related to spin 1. The value 1776.91 MeV will be discussed in (36) with a reference. A very surprising relation between Quarks and Leptons with the same $\frac{1}{1-c} \cong 44.63955017596401$ as in (22) is the relation between the **pole energy of the Bottom/Beauty Quark** [23], [24] and the anti-**Muon**, this time we take the Muon value that yields in (24) along with the denominator of (23), the exact mass ratio between the Muon and the electron $105.65837455\ MeV$ instead of the 2014 value $105.6583745\ MeV$,

$$\frac{105.65837455\ MeV}{(1-c)(1+(a-1)(1-b))}\left(1+\sqrt{(c_1-1)(1-c_2)}\right)$$
$$= 44.63955017596401 * 105.65837455\ MeV$$
$$* (1+75.37831150257286827786078900969 3^{-1}) *$$
$$(1+12202.88874066467724^{-1})^{-1} \cong 4{,}778.72231644255851132 99 \text{ MeV} \approx \mathbf{4.78\ GeV}$$

Which is equivalent to:

$$\frac{PoleEnergyOfBottomQuark*(1-c)}{(1+\sqrt{(c_1-1)(1-c_2)})} = \frac{MuonEnergy}{(1+(a-1)(1-b))} \tag{33}$$

In which the root in the left denominator is attributed to spin 1. New physics? Looks like it! A.M. Badalian's prediction 4,778 MeV [24] is too close to 4,778.72 MeV to be ignored. The outcome of the Muon being the electro-gravitational energy of the pole energy of the Bottom Quark is as follows:

a) Lepton universality should be broken in decays of anti-Bottom Quark that involve Muons.

b) High energy p-p collisions can no longer be considered for the calculation of the W Boson mass.

Before we proceed, it is worthy of mentioning the following Simon Plouffe identity [25]: consider the functions, $S_n(r) = \sum_{k=1}^{\infty} \frac{1}{k^n e^{\pi r k}-1}$ then there is a well-known relation between π and 96, $\pi = 72S_1(1) - 96S_1(2) + 24S_1(4)$. Notice that the sum of the positive coefficients 72 + 24 = 96 and the negative coefficient is -96. While this identity is not a direct relation between $\xi = \frac{4}{\pi}$ and $\xi = \frac{95}{96} = 1 - \frac{1}{96}$, it does show an example of how π and 96 can be related to each other through Zeta functions in a simple and straight forward manner. A deeper and a very surprising relation will be seen in a note after (40).

7. The exact inverse Fine Structure constant – critical imbalance between gravity and anti-gravity

The following endeavor originated in the search for a field strength coefficient near $\frac{\pi}{2}$ for quite a simple reason. If a motion in a small circle is with the constant velocity c, then after half a circle the velocity will be -c. The difference c-(-c) is 2c and the time between the two velocity measurements is $\frac{\pi r}{c}$ so $2c(\frac{\pi r}{c})^{-1} = \frac{2}{\pi}\frac{c^2}{r}$ while the acceleration of the motion is $\frac{c^2}{r}$. The inferred acceleration $\frac{2}{\pi}\frac{c^2}{r}$ can be interpreted only when the velocity can take one of two values c or -c, or in other words when velocity itself is quantized. The correction in this situation is by a factor $\frac{\pi}{2}$ and $\frac{\pi}{2}\frac{2}{\pi}\frac{c^2}{r} = \frac{c^2}{r}$. Given a radius r and an upper speed limit c, the correction coefficient $\frac{\pi}{2}$ should be considered as a possible upper field strength coefficient. The way a coefficient near $\frac{\pi}{2}$ was found will be discussed along with its relation to the inverse Fine Structure Constant. The fine structure constant is surprisingly reached through the mass ratio between the Tau lepton and the Muon and an interesting perturbation of the field strength of the Tau lepton that will be found in this section. Recommended reading for this section is Appendix E, (70) - (79).

Note: The more advanced parts of this section require basic knowledge of electrical engineering and especially a good understanding of the trivial subject of Dissipation Factor and Loss Tangent and especially of Power Factor [26].

Note: Why dissipation factor? The reason is that any perturbation of the Reeb field, which behaves as acceleration, above a maximal allowed limit, must be emitted and in mainstream physics, the electromagnetic field is dissipated as photons.

The denominator $1 + \sqrt{(c_1 - 1)(1 - c_2)}$ in (32), (33) and $(1 + (a - 1)(1 - b))$ in (24) can be used together to yield a nice result that seems to be more than just a mathematical coincidence. Consider the following imbalance equation as in (23) of negative and positive charge:

$$1 + \frac{1}{96}\left(-\frac{1}{2}\xi^2 {g_1}^{-2} + \xi {g_1}^{-1}\right) = g_1$$
$$1 + \frac{1}{96}\left(-\frac{1}{2}\xi^2 {g_2}^{-2} - \xi {g_2}^{-1}\right) = g_2$$

$$\text{Such that } (g_1 - 1)^{-\frac{1}{2}} = \frac{1}{2}(1 - g_2)^{-1} \qquad (34)$$

With biggest roots g_1 ≅1.003629541 and g_2 ≅0.969877163. g_1 means an area portion $\sim\mathbf{275.51693^{-1}}$ is added around a negative charge and $\sim\mathbf{33.19740^{-1}}$ of the area is subtracted around a positive charge, which reflects a possibly maximal allowed gravitational imbalance between negative and positive charge.

A calculation that uses an electronic datasheet, yields,

$$\xi \cong \mathbf{1.5561985371903484},\ (g_1 - 1)^{-\frac{1}{2}} \cong \mathbf{16.59870203} \qquad (35)$$

which is close to the known mass ratio between the Tauon and the Muon, ≅16.817 where ξ denotes a maximal allowed coefficient. Multiplying this value by

$1+\sqrt{(c_1-1)(1-c_2)}$ from (32), (33) and dividing by $\left(1+(a-1)(1-b)\right)$ from (24) yields,

$$\frac{Muon\ 105.6583745\ MeV}{\left(1+(a-1)(1-b)\right)} \cong \frac{\sqrt{g_1-1}\ Tauon\ 1776.9127923826\ MeV}{\left(1+\sqrt{(c_1-1)(1-c_2)}\right)} \tag{36}$$

Which is ≅16.81752914. So, this calculation predicts a Tauon energy of about **1776.9127923826 MeV** which agrees with [27]. Please note the remark after (28.1) for a possible additional W Boson. We now need to check the consistency of (36) with (32) as a test to this theory. We take $\frac{1}{1-x_2}$ from (22) and $\frac{1}{\sqrt{(c_1-1)(1-c_2)}}$ from (28) and check the following:

$$\frac{1776.91279322344\ldots\text{MeV}*(1+\sqrt{(c_1-1)(1-c_2)})}{1-x_2} \cong 80372.8876666694MeV \tag{36.1}$$

Which is consistent with (32) but less with (32.1) of a higher W Boson energy as the approximation of the W Boson's energy with $\xi=\frac{4}{\pi}$ and a null Reeb vector. For $\xi=\frac{4}{3}$ the W Boson energy is a bit higher. (35) is strikingly related to (20) and (22). $\frac{192{y_1}^2+2\left(\frac{4}{\pi}\right)y_1-\left(\frac{4}{\pi}\right)^2}{192}={y_1}^3$ and $\frac{192{y_1}^2-2\left(\frac{4}{\pi}\right)y_2-\left(\frac{4}{\pi}\right)^2}{192}={y_2}^3$ in the following way:

Assessing the following yields,

$$-\frac{1}{\log(y_1)}\frac{1}{\log(y_2)} \cong 9147.5718747432856616796925 66 \tag{36.2}$$

and on the other hand from (35),

$$\frac{1}{(g_1-1)}\frac{1}{(1-g_2)} \cong 9146.4461480441150342812761 66 \tag{36.3}$$

The relative error in these two values in relation to $\frac{1}{(g_1-1)}\frac{1}{(1-g_2)}$ is Relative error ≅ $8124.92601871057195239700377 0^{-1}$. Please note that for a small d the following holds. $\frac{1}{d}\approx\frac{1}{\log(d+1)}$ and also $\frac{1}{d}\approx-\frac{1}{\log(1-d)}$. This relation alludes to a possible exponential relation between the roots of (20), (22) and the roots of (35) but before we actually check an exponential perturbation on the field strength ξ ≅1.5561985371903483965638770314399 from (35) we notice the following for the same field strength coefficient of (35):

$$\frac{2}{\cos(\xi)} \cong \frac{2}{\cos(1.5561985371903484)} \cong 137.011909869, \tag{37}$$

$$\tan^{-1}(95^2 96^2(1-g_2)^{+4}) \cong 1.5561948778250207190765973767615 \tag{38}$$

remarkably approximate ξ ≅1.5561985371903484 from (34), (35).

$$Error=\frac{\xi-\left(95^2 96^2(1-g_2)^4\right)}{\xi} \cong 425{,}263.601328167905179588241571 33^{-1} \tag{39}$$

In terms of electrical engineering Dissipation Factor and Loss Tangent, we can write, $DF = \frac{95^2 96^2}{(1-g_2)^{-4}} \approx \tan(\xi)$ where the numerator is known as the Resistive Power Loss and the denominator as the Reactive Power Oscillation. It is expected that an oscillating charge will generate oscillation in area due gravity changes, however, it is not expected that the area portion that is lost due to gravity will appear as the power of 4. This is a very rare property that connects between trigonometry and the electro-gravity polynomials (34). We can get from this relation two insights, the first is that if (37) is not a mathematical coincidence, then the inverse Fine Structure constant should come out of a trigonometric function and a numbers relation. The second is that $95^2 96^2 (1-g_2)^4$ should be part of this equation. We may think that perhaps scaling of the value of ξ in a rational way, will yield the exact inverse Fine Structure Constant. So we want to find some d such that $\frac{2}{\cos\left(1.5561985371903484*(1+\frac{1}{d})\right)}$ will yield the constant we are looking for. We will soon find such d, $d \cong 606400.8$ that complies with [28] and we get, $\frac{2}{\cos\left(1.5561985371903484*(1+\frac{1}{606400.8})\right)} \cong 137.0359990462475253$. The motivation for this endeavor is taken from electrical engineering [26] where the cosine term means a ratio between delivered power and actually measured power in motors and other electric devices. In our case, we are interested in the ratio between radiation's energy and the energy it delivers upon interaction.

Until now, d is not very interesting because we could not find d out of any new theory. Well, not very accurate. First, $\frac{1}{2}(1-g_2)^{-4} \cong 607276.5368006824282929 \approx 606400.8$ and $\frac{2}{\cos(1.5561985371903484*(1+2(1-g_2)^4))} \cong 137.0359643018112763$

If we test the following values for $d \cong 606400.8$ we get: $\frac{95^4}{d} \cong 134.3181357940161..$, $\frac{96^4}{d} \cong 140.0635619214..$ and the geometric average of these two values is $\left(\frac{95^4}{d}\frac{96^4}{d}\right)^{\frac{1}{2}} = \frac{95^2 96^2}{d} \cong 137.1607689$. It is not difficult to see the following:

As a result of the conclusions of (38), (39), the exact inverse Fine Structure Constant was found by the following, although some aspects of the following calculation are not resolved yet. We put together (20), (22), (34), (35), (37), $\frac{1}{2}(1-g_2)^{-4} \cong 607276.5368006824282929$, and from (35) $\xi \cong 1.5561985371903484$

$$1 + \frac{1}{96}\left(-\frac{1}{2}\left(\frac{4}{\pi}\right)^2 a^{-2} + \frac{4}{\pi}a^{-1}\right) = a \Rightarrow \frac{1}{a-1} \cong 206.75133988502202$$

$$1 + \frac{1}{96}\left(-\frac{1}{2}\left(\frac{4}{\pi}\right)^2 b^{-2} - \frac{4}{\pi}b^{-1}\right) = b \Rightarrow \frac{1}{1-b} \cong 44.63955017596401$$

$$d = \left(\frac{1}{2}(1-g_2)^{-4}\right)^{\frac{1}{1+(a-1)(1-b)}} \cong \mathbf{606401.0372} \approx \mathbf{606400.8}$$

$$\frac{2}{\cos\left(1.5561985371903484*(1+\frac{1}{d})\right)} \cong \mathbf{137.0359990368270076} \approx \mathbf{137.035999037} \qquad (40)$$

$1.5561985371903483965638770314399 * (1+\frac{1}{d})$ exceeds the maximal allowed value of the field strength $\xi = 1.5561985371903483965638770314399$ and therefore must account for emission of what we know in mainstream physics as photons.

Note: $p = ((a-1)(1-b))^{-\frac{1}{2}} \cong 96.0691772148863$ is a very special number in the following property that bridges between area ratios and powers as follows, denote $s = \frac{1}{2}(1-g_2)^{-4}$ then $s^{(\frac{1}{1+(a-1)(1-b)})} \approx s(2-\frac{1}{96^2(a-1)(1-b)})$ or written as numbers 606401.0372 ~ 606401.0194 with a relative error of about 34,109,836.56^{-1}. An exact equality, $s^{\frac{1}{1+p^{-2}}} = s\left(2-\frac{p^2}{96^2}\right) \cong 606401.0371$, follows from replacing $p = \sim 96.0691772148863$ with p $= \sim 96.06917582$ with a relative error in $96.0691772148863 = ((a-1)(1-b))^{-1/2}$ of ~1.45953 * 10^{-8}. If the reader still thinks (40) is a fluke of chance, then this note does not agree with such a hypothesis. Also note that p comes from (20), (22) which resulted in (24). See Python code and it's more exact output in Appendix F.

Another result is by finding the variable s where a and b are given in (40):

$$\left(\frac{95^2*96^2}{s}\right)^{1+(a-1)(1-b)} = \frac{2}{\cos\left(\xi\left(1+\frac{1}{s^{\left(\frac{1}{1+(a-1)(1-b)}\right)}}\right)\right)} \Longrightarrow \qquad (41)$$

$$\left(\frac{95^2*96^2}{s}\right)^{1+(a-1)(1-b)} \cong \mathbf{137.035999036428876252}$$

If we search for s and replace $(a-1)(1-b)$ with $(96*95)^{-1}$ we get **137.0359992990990**, $s \cong$607280.4243559269234538 and $s^{1/(1+(96*95)^{-1})} \cong 606394.43614689458627253770$

Another idea is to solve the following equation where s is given by (34) and p is a variable:

$$s = (\frac{1}{2}(1-g_2)^{-4}) \cong 607276.536800682428292930$$

$$\left(\frac{95^2*96^2}{s}\right)^{1+1/(p*p)} = \frac{2}{\cos\left(\xi\left(1+\frac{1}{s^{\left(\frac{1}{1+1/(p*p)}\right)}}\right)\right)} \Longrightarrow \qquad (42)$$

$$\left(\frac{95^2*96^2}{s}\right)^{1+1/(p*p)} \cong \mathbf{137.035999035747181551}, p \cong \mathbf{96.070666670305840285}$$

For comparison, if we set p=96 in the right-hand side of (42) we get the value **137.035999086935760260530515**. Combining (41) and (42) we find a numerical attractor at (42) with $s \cong \mathbf{607276.53680068242829293012 62} \cong (\frac{1}{2}(1-g_2)^{-4})$, $s^{\left(\frac{1}{1+1/(p*p)}\right)} \cong$ $\mathbf{606401.064296812633983791}, \xi \cong \mathbf{1.5561985371903484}$ from (35). Before we close this discussion, it is nice to mention another relation $(1-\ln\left(\left(1+\frac{1}{137.035999035747181551}\right)^{137.035999035747181551}\right))^{-1} \cong 275.4045237287 \approx 275.51693 \cong$ $(g_1-1)^{-1}$ in (43.10). That is not a total surprise because $(1-\ln\left(\left(1+\frac{1}{z}\right)^{z}\right))^{-1} \approx 2z$ for big z.

Reverse engineering Nature – Looking for simple but not random relations

In this section a much less significant result than (24), (40), remark after (40), (41), (42), will be considered as an interesting course of research. This time, an approximation of the inverse Fine Structure Constant will not be as nearly as accurate and will not be a result of exponential perturbations of a Reeb field strength.

The search for meaningful field strength coefficients for the electron, Muon and Tau lepton reached the following $\xi \in \{\frac{95}{96}, \frac{4}{\pi}, \sim 1.556198537190348396563877031439 9\}$

But these field strength coefficients did not appear out of solutions to equation (4). In fact, there has been no collaboration with mainstream physics to reach such solutions and especially to the complex form of (4). The analytic solutions of such an equation make take decades and without collaboration on solving the Lagrangians in (4), (64), (65), other approaches are required in order to convince the reader that the choices of field strength coefficients are not a mere mathematical pareidolia. The assessment of the mass ratio between the Muon and the electron in (24) is already with a sufficiently small error to trigger interest, especially when considering the simplicity of (24) and that the choice of $\frac{4}{\pi}$ came out of an existing theory [20]. (40), the remark after (40), (41) and (42) are also strong indicators that this research is on the right path. It will be wrong not to mention other findings which are straight forward from the method which had been presented in (16), (17), (18), (19) and the first interesting result (20). In this method, the Reeb vector term was collapsed with the non-geodesic or accelerated time direction $\frac{P^\mu}{\sqrt{|Z|}}$ and we saw the contraction $\frac{1}{4}\left(U_\mu U_\nu - \frac{1}{2} g_{\mu\nu} U_\lambda U^\lambda - 2U^k;_k \frac{P_\mu P_\nu}{Z}\right)\frac{P^\mu P^\nu}{Z}$ that resulted in (20), (22).

With acceleration field $\frac{\xi c^2}{r}$, where $\xi = \frac{4}{\pi}$ denotes the field strength and x is the adjustment factor of the acceleration field because of area loss, we used the term $\left(-\frac{1}{2}\frac{\xi^2}{r^2 x^2} \mp \frac{\xi}{r^2 x}\right)\frac{\pi}{24} r^4 =$

$\left(-\frac{1}{2}\frac{\xi^2}{x^2} \mp \frac{\xi}{x}\right)\frac{\pi}{24}r^2$ to express area loss due to a gravitational field at small r in the far observer coordinates. Now it is time to look at area loss in a direction perpendicular to the direction of time, namely the momentum direction in spacetime, or as expressed through a bivector derived from a unit vector, consider $\frac{U^\mu U^\nu}{U^\lambda U_\lambda}$ and for the sake simplicity, the contraction is not with a complex bivector $\frac{2U^{*\mu}U^{*\nu}}{U^{*\lambda}U_\lambda + U^\lambda U^{*}{}_\lambda}$. From $U^\mu P_\mu = 0$, it is easy to see the following,

$$\frac{1}{4}\left(U_\mu U_\nu - \frac{1}{2}g_{\mu\nu}U_\lambda U^\lambda - 2U^k{}_{;k}\frac{P_\mu P_\nu}{Z}\right)\frac{U^\mu U^\nu}{U^\lambda U_\lambda} = \frac{1}{8}U_\lambda U^\lambda = \frac{1}{2}\frac{\xi^2}{r^2 x^2} \tag{42.1}$$

The latter is to achieve a reduction of the curvature calculation from Lorentzian to Riemannian geometry.

Caveat: Notice that using $\frac{U^\mu U^\nu}{U^\lambda U_\lambda}$ and not $\frac{U^\mu U^\nu}{|U^\lambda U_\lambda|}$ is done here in order to achieve $g_{\mu\nu}\frac{U^\mu U^\nu}{U^\lambda U_\lambda} = +1$ as expected from a unit vector in (+,-,-,-) metric convention. The reader may criticize this choice of a bivector because Reeb vectors in this paper are space-like and not time-like because they represent non-geodesic acceleration as a result of misaligned events in an observer spacetime object.

Multiplying by $\frac{\pi}{24}r^4$ due to [18], see lecture of Seth Lloyd, and dividing by 4 times the area of an Euclidean disk, due to assumption 2 after the note after (15), yields,

$$-\frac{1}{192}\frac{\xi^2}{{x_3}^2} = \frac{1}{2}\frac{\xi^2}{r^2 {x_3}^2}\frac{\pi}{24}r^4\frac{1}{4\pi r^2} = x_3 - 1 \tag{42.2}$$

From which

$$\frac{192{x_3}^2 - \xi^2}{192} = {x_3}^3 \tag{42.3}$$

Which is an iterative equation that converges to the most stable root, a technique that had been used in all previous third order polynomial equations. Solving for $\xi = \frac{4}{\pi}$ as in (20), (22), yields,

$$(x_3 - 1)^{-1} \cong 120.41061116112391982824192382395267486572265625 \tag{42.4}$$

$$(x_1 - 1)^{-1} \cong 206.7513398850220198710303520783782005310058593 75$$

$$(1 - x_2)^{-1} \cong 44.63955017596401120272275875322520732879638671875$$

And the following calculation yields an interesting result,

$$\ln\left((x_1 - 1)^{-1}(1 - x_2)^{-1}(x_3 - 1)^{-1}\right)^2 2^{-\frac{1}{2}}$$

$$\cong \ln(1111304.0650477090384811162948608398437)^2\, 2^{-\frac{1}{2}} \cong$$

$$137.0341023246677139013627311214804649353 \qquad (42.5)$$

Which is a surprisingly simple and unexpected approximation of the inverse Fine Structure Constant. The relative error of (42.5) in relation to the result in (40) is about 72249.23316^{-1} which is not even closely significant as (40), (41), (42) or the remark after (40) and yet, if this result joins other approximations of the inverse Fine Structure Constant in this paper, it is not wise to ignore (42.5). In (24.3), (40), (41), (42), (81)-(86), the inverse Fine Structure Constant comes out of exponential field perturbations as in (24.3), (40), (41), (42) or as exponential functions of 2 or $\frac{4}{\pi}$ with coefficients $(95*96)^{-1}$ or 95 and 96 as seen in (81)-(86). Notice that both (24.3) and the last result, involve the square root of 2.

Hypergeometric tests - Dr. Sam Vaknin's suggestion from 2013

A suggestion from Dr. Sam Vaknin regarding the possible solutions of the equations of the Geometric Chronon Field Theory was that they are related to Hypergeometric functions [29]. His idea was lately checked regarding the stable roots of third order polynomials of gravity and anti-gravity, area ratio loss and gain, see (22.1) and (22.2). The stable field strength coefficients were defined as $\xi = \frac{193}{192} = 1 + \frac{1}{192}$ for negative charge and $\xi = \frac{63}{64} = 1 - \frac{1}{64} = 1 - \frac{3}{192}$ for positive charge. The summation of the two deltas $+\frac{1}{192} - \frac{1}{64}$ to 1 yields the field strength coefficient $\xi = 1 + \frac{1}{192} - \frac{1}{64} = 1 + \frac{1}{192} - \frac{3}{192} = 1 - \frac{1}{96} = \frac{95}{96}$. The question is what do these values $\frac{1}{192}$ and $\frac{3}{192}$ teach us about any possible grand theory of particle physics? 1,3 and 192 with 192 in the denominator should hint us about such a theory. As we saw in (40), (41), (42) a key number in the calculation of the positive perturbation over ξ was

$s = 0.5/(1 - g_2)^4 \cong 607276.5368006824282929301261901855468 75$, see (42).

Can this number be a result of combinatorial mixing by the Gauss hypergeometric function 2F1?

The question is if ${}_2\mathrm{f}_1(\mathrm{a, b, c, z}) = \sum_{k=0}^{\infty} \frac{(a)_n (\mathrm{b})_n}{(c)_n} \frac{z^n}{n!}$, such that $(q)_n = \left\{ \begin{matrix} 1 \\ q(q+1)(q+2)\dots(q+n-1) \end{matrix} \middle| \begin{matrix} n = 0 \\ n \geq 1 \end{matrix} \right\}$ can yield such a number in a meaningful way.

If Dr. Sam Vaknin was right, we may be able to find a meaningful z that solves

$${}_2\mathrm{f}_1(-3,1,192,\mathrm{z}) = 1 - 2(1 - g_2)^4 \qquad (42.6)$$

This is exactly what was done numerically. The result was very surprising, and it is very unlikely that it is a fluke of chance:

$$z \cong \frac{2}{(137.036271402616947057140350807 4)^2} \tag{42.7}$$

The relative error of $137.0362714026169470571403508074$ from the assessment 137.035999036827007557 8… in (40) is about $\frac{137.0359990368270075578 - 137.0359990368270075578\ldots}{137.0359990368270075578\ldots} \cong 503132.19978307740529999136 92^{-1}$

Also, quite near the higher value 137.0359992990990… after (41).

It is quite compelling to say that Dr. Sam Vaknin was right already back then in 2013. There is even stronger evidence in his favor.

Consider a second order perturbation on the hypergeometric coefficients -3, 1:

$$_2f_1\left(-3 * \frac{63}{64}, 1 * \frac{193}{192}, 192, \frac{2}{(137.0359990368270075578\ldots)^2}\right) \cong$$

$$1 - \frac{1}{607299.79207959282211959362030029296875 0} \tag{42.8}$$

and

$$_2f_1\left(-3 * \frac{193}{192}, 1 * \frac{63}{64}, 192, \frac{2}{(137.0359990368270075578\ldots)^2}\right)$$

$$\cong 1 - \frac{1}{607299.8060423402348533272743225097656 25} \tag{42.9}$$

Comparing the right hand side denominator to $\frac{1}{2}\frac{1}{(1-g_2)^4} \cong$ $607276.5368006824282929301261901855468 75$ from the remark before (40) and from (42), the results in (42.8) and (42.9) are very interesting although not within the ranges of (40)-(42) with a highest value of 137.0359992990990… .

Instead of $\frac{63}{64}$ if we consider all the powers of $-\frac{1}{64}$ we have $\frac{64}{65} = \sum_{k=0}^{\infty}\left(-\frac{1}{64}\right)^k$ and with $+\frac{1}{192}$ we have $\frac{192}{191} = \sum_{k=0}^{\infty}\left(\frac{1}{192}\right)^k$

Consider a second order perturbation on the hypergeometric coefficients -3, 1:

$$_2f_1\left(-3 * \frac{64}{65}, 1 * \frac{192}{191}, 192, \frac{2}{(137.0359990368270075578\ldots)^2}\right) \cong$$

$$1 - \frac{1}{607135.0557247012620791792869567871093 75} \tag{42.8.1}$$

and

$$
{}_2f_1\left(-3 * \frac{192}{191}, 1 * \frac{64}{65}, 192, \frac{2}{(137.035999036827007557 8\ldots)^2}\right)
$$

$$
\cong 1 - \frac{1}{607135.069557101931422948837280273437500} \tag{42.9.1}
$$

Here is the code in Python for (42.6) and (42.7):

```
import numpy as NP
from scipy.special import hyp2f1 as SCIPY_SPECIAL_hyp2f1

a = 137.035999036827007557803881354629993438720703
q = 607276.536800682428292930126190185546875

#s = NP.power(q * 2, 0.25)
s = NP.sqrt(NP.sqrt(q * 2))
s = NP.sqrt(s * s * s * 0.25)

print(f's={s:.42f}')

# Was a numerical analysis output:
w = 137.036271402616947057140350807 4
u = 1/(w/a - 1)

print(f'u={u:.42f}')

r = SCIPY_SPECIAL_hyp2f1(-3, 1, 192, 2/(w ** 2))
r = 1/(1-r)
r /= 607276.536800682428292930126190185546875
r = 1/(1-r)
r /= s
print(f'r={r:.42f}')


r = SCIPY_SPECIAL_hyp2f1(-3, 1, 192,
2/137.035999036827007557803881354629993438720703 ** 2)
r = 1/(1-r)
r /= 607276.536800682428292930126190185546875
r = 1/(1-r)
print(f'r={r:.42f}')
```

Here is the code in Python for (42.8), (42.9)

```
import numpy as NP
from scipy.special import hyp2f1 as SCIPY_SPECIAL_hyp2f1

Xi = 1.556198537190348396563877031439915299415588378906

r1 = \
    SCIPY_SPECIAL_hyp2f1(-3 * 63/64, 1 * 193/192, 192,
                         2/137.035999036827007557803881354629993438720703 **
                         2)
r1 = 1/(1-r1)
```

```
print(f'1/(1-r1)={r1:.33f} compared to'
      f' 607276.536800682428292930126190185546875')

inverse_alpha1 = 2/NP.cos(Xi*(1+1/NP.power(r1, 1/(1+1/(95*96)))))
print(f'Inverse alpha(r1) {inverse_alpha1:.33f}')

r2 = \
    SCIPY_SPECIAL_hyp2f1(-3 * 193/192, 1 * 63/64, 192,
                         2/137.035999036827007557803881354629993438720703 **
                         2)
r2 = 1/(1-r2)
print(f'1/(1-r2)={r2:.33f} compared to'
      f' 607276.536800682428292930126190185546875')

inverse_alpha2 = 2/NP.cos(Xi*(1+1/NP.power(r2, 1/(1+1/(95*96)))))
print(f'Inverse alpha(r2) {inverse_alpha2:.33f}')

r = r1 / r2
r = 1/(1-r)
print(f'1/(1-r1/r2)={r:.33f}')

r3 = \
    SCIPY_SPECIAL_hyp2f1(-3 * 64/65, 1 * 192/191, 192,
                         2/137.035999036827007557803881354629993438720703 **
                         2)
r3 = 1/(1-r3)
print(f'1/(1-r3={r3:.33f} compared to'
      f' 607276.536800682428292930126190185546875')

inverse_alpha3 = 2/NP.cos(Xi*(1+1/NP.power(r3, 1/(1+1/(95*96)))))
print(f'Inverse alpha(r3) {inverse_alpha3:.33f}')

r4 = \
    SCIPY_SPECIAL_hyp2f1(-3 * 192/191, 1 * 64/65, 192,
                         2/137.035999036827007557803881354629993438720703 **
                         2)
r4 = 1/(1-r4)
print(f'1/(1-r4)={r4:.33f} compared to'
      f' 607276.536800682428292930126190185546875')

inverse_alpha4 = 2/NP.cos(Xi*(1+1/NP.power(r4, 1/(1+1/(95*96)))))
print(f'Inverse alpha(r4)
{2/NP.cos(Xi*(1+1/NP.power(r4,1/(1+1/(96*96))))):.33f}')

r = r3 / r4
r = 1/(1-r)
print(f'1/(1-r3/r4)={r:.33f}')

print(f'(1/Alpha1+1/Alpha3)/2: '
      f'{(inverse_alpha1+inverse_alpha3)/2:.33f}')
print(f'(1/Alpha2+1/Alpha4)/2: '
      f'{(inverse_alpha2+inverse_alpha4)/2:.33f}')
```

8. The mass hierarchy

By (13) and considering the Planck mass $\sqrt{\frac{\hbar c}{K}}$ and the Fine structure constant Alpha:

$$\sqrt{\frac{\hbar c}{K} * \frac{e^2}{4\pi\varepsilon_0 \hbar c}} = \frac{2e}{2\sqrt{4\pi K\varepsilon_0}} = \frac{2e}{\sqrt{16\pi K\varepsilon_0}} = PlanckMass * \sqrt{Alpha} \tag{43}$$

So, multiplication of the Plank mass by the square root of the Fine Structure Constant yields twice the electro-gravitational mass of a charge e! If we take $\xi \cong 1.5561985371903484$ from (35) to be the maximal allowed field coefficient of an electric charge, then the field around a single charge as a normalized quantity is obtained as

$$\frac{1}{\xi}\frac{1}{2} PlanckMass * \sqrt{Alpha} = \frac{1}{\xi}\frac{e}{\sqrt{16\pi K\varepsilon_0}} \tag{44}$$

Now we recall from (24) the following root a around a negative charge:

$$1 + \frac{1}{96}\left(-\frac{1}{2}(\frac{95}{96})^2 a^{-2} + \left(\frac{95}{96}\right) a^{-1}\right) = a \cong 1 + 192.0463944^{-1} \tag{45}$$

We take from (24), (40), $(a-1)(1-b) \cong \frac{1}{206.75133988502202 * 44.63955017596401}$ and calculate

$$\left(\frac{\frac{1}{\xi}\frac{1}{2} PlanckMass * \sqrt{Alpha}}{M_e}\right)^{(a-1)(1-b)} \cong 1 + 192.04864774452^{-1} \tag{46}$$

Where $M_e \cong 0.5109989461\ MeV$, e is the electron's charge 1.602176634×10^{-19} Coulombs, K is Newton's constant of gravity 6.674×10^{-11} $m^3 \cdot kg^{-1} \cdot s^{-2}$, Planck mass 1.22091×$10^{22}$ MeV, from (40) $Alpha \cong 137.0359990368270076^{-1}$. The relative error between (46) and (45) is $\frac{192.04864774452 - 192.0463944}{192.0463944} \cong 85{,}227.266539382^{-1}$. We are also led to the following conclusion that $\xi = \frac{95}{96}$ is the Reeb field strength coefficient of the electron field, $\xi = \frac{4}{\pi}$ is the Muon field strength and from the solution to (35) $\xi = 1.5561985371903484$ ... is the field strength coefficient of the Tau lepton. Of course, a lot of work has to be done to achieve exact analytic solutions to (4) and as we shall see also to (64), because only $\xi = \frac{4}{\pi}$ has a compelling source [20]. Serendipity is part of physics and mathematical rigor must follow.

9. Interesting acceleration to radius coefficients relation – the field strength coefficients

Consider the coefficients $\frac{95}{96}$, from (23) $\frac{4}{\pi}$, from (24) and $\xi =$ 1.556198537190348396563877031439915299415588 from (34), (35). Note the following table

ξ of Electron, Muon, Tau	$\xi\left(\frac{4}{\pi}\right)^{-1}$	$\xi \cdot 9 \cdot \left(\frac{4}{\pi}\right)^{-1}$

$\frac{95}{96}$	$0.7772169325287248897 \sim \frac{7}{9}$	6.994952392758524
$\frac{4}{\pi}$	$1 = \frac{9}{9}$	9
1.5561985371903483965638770314399	$1.22223547299109529 \sim \frac{11}{9}$	11.0001192569

The reader can check that 7 * 9 * 11 = 693 is the integer floor of $\left(\frac{1}{96^2(a-1)(1-b)} - 1\right)^{-1} \cong 693.634239847$, see the note after (40) with $2 - \frac{1}{96^2(a-1)(1-b)}$. We have yet to show more compelling evidence the choice of ξ is not by chance. Some readers will remain skeptical no matter what evidence is brought in this paper. This section is not meant for such readers but for readers who agree that serendipity is important for new discoveries in physics. The coefficient $\frac{4}{\pi}$ from (22), (80) is well understood [20], however, $\frac{95}{96}$ from (23), (79), (86) and 1.5561985371903483965638770314399 from the solution to (35) are not well understood. Evidence, except from the previous table and the note after (40), can be found if we look at the polynomial term that means loss or addition of area in relation to 4 times the area of a disk. The factor 4 was thoroughly discussed before (16) and led to the number 96 from $\frac{\frac{1}{4}*\frac{\pi}{24}*R(3)*r^4}{\pi r^2} = \frac{R(3)*r^2}{96}$ where R(3) is obtained by double contraction of the Einstein tensor with a direction of time. We return to (18), $\left(-\frac{1}{2}\frac{\xi^2}{x^2} \mp \frac{\xi}{x}\right)\frac{1}{96} = \frac{\delta Area}{4\pi r^2}$ and consider the following polynomials $\left(-\frac{1}{2}\xi^2 \mp \xi\right)\frac{1}{96}$ of the field strength coefficient ξ. Like before in (23), we consider the terms $\frac{1}{a-1}$ and $\frac{1}{1-b}$ from the biggest and stable roots a, b. Not too surprisingly, these terms are approximated by $\alpha = p1(\xi) = \left(\left(-\frac{1}{2}\xi^2 + \xi\right)\frac{1}{96}\right)^{-1}$ and $\beta = p2(\xi) = \left(\left(-\frac{1}{2}\xi^2 - \xi\right)\frac{1}{96}\right)^{-1}$ which only depend on the field strength coefficients. Consider the following relative error terms $RatioA = \left(\frac{a-1}{\alpha} - 1\right)^{-1}$ and $RatioB = \left(\frac{1-b}{\beta} - 1\right)^{-1}$ or as an output of a python code:

```
Field strength coefficient analysis:

Xi=0.9895833333333334, p1=192.02083559413998, p2=64.89902805794975

Xi=0.9895833333333334, 1/(a-1)=192.04639436012951, 1/(1-b)=63.54135822920768

Xi=0.9895833333333334, RatioA=-7513.91496909199486, RatioB=46.80177527998884

-----------------------------------------------------

Xi=1.2732395447351628, p1=207.49126659259227, p2=46.06948110927548

Xi=1.2732395447351628, 1/(a-1)=206.75133988502202, 1/(1-b)=44.63955017596401
```

```
Xi=1.2732395447351628, RatioA=279.42137750905704, RatioB=31.21797643232097

-----------------------------------------------------

Xi=1.5561985371903484, p1=278.00172875202145, p2=34.69366870085835

Xi=1.5561985371903484, 1/(a-1)=275.51690891864394, 1/(1-b)=33.19740405023536

Xi=1.5561985371903484, RatioA=110.88003452715024, RatioB=22.18685313217340

-----------------------------------------------------
```

This output shows proximities between functions of the field strength coefficient, ξ or Xi in the Python output. The proximities are p1 of the next ξ to RatioA and p2 of the next ξ to RatioB. The first value ~-7513.91496909199486 is in red as an exception because it is not matched to the value of p1 for the next field strength coefficient $\xi = \frac{4}{\pi}$. Following is the code in Python that was used for the last calculations,

```
import numpy as NP


def function_p(p_x):

    return (-0.5 * p_x * p_x + p_x)/96, -(-0.5 * p_x * p_x - p_x)/96


def function_cubic_viete(a, b, c, d): # If all roots are real.

     # Viete's formula when all roots are real.

     b2 = NP.longdouble(b * b)

     b3 = NP.longdouble(b2 * b)

     a2 = NP.longdouble(a * a)

     a3 = a2 * a

     p = (3 * a * c - b2) / (3 * a2)

     q = (2 * b3 - 9 * a * b * c + 27 * a2 * d) / (27 * a3)

     offset = b / (3 * a)

     t1 = 2 * NP.sqrt(-p / 3) * NP.cos(NP.arccos(NP.sqrt(-3 / p) \

                                                  * (3 * q) / (2 * p)) / 3)

     t2 = 2 * NP.sqrt(-p / 3) * NP.cos(NP.arccos(NP.sqrt(-3 / p) * \

                                                  (3 * q) / (2 * p)) / 3 - NP.pi / 3)

     t3 = 2 * NP.sqrt(-p / 3) * NP.cos(NP.arccos(NP.sqrt(-3 / p) * \
```

```
                                            (3 * q) / (2 * p)) / 3 - 2 * NP.pi /
3)

     x1 = t1 - offset

     x2 = t2 - offset

     x3 = t3 - offset


     return (x1, x2, x3)


ma_list = [95/96, 4/NP.pi, 1.5561985371903484]


print('Field strength coefficient analysis:')


for ma_x in ma_list:

    ma_tuple = function_p(ma_x)


    ma_a,_,_ = function_cubic_viete(1, -1, -ma_x / 96,

                                    (ma_x * ma_x) / 192)


    ma_b,_,_ = function_cubic_viete(1, -1, ma_x / 96,

                                    (ma_x * ma_x) / 192)


    print('Xi={}, p1={:.14f}, p2={:.14f}'.format(ma_x, 1/ma_tuple[0], 1/ma_tuple[1]))

    print('Xi={}, 1/(a-1)={:.14f}, 1/(1-b)={:.14f}'.format(ma_x, 1/(ma_a-1), 1/(1-
ma_b)))


    ma_a = (ma_a - 1) / ma_tuple[0]

    ma_b = (1 - ma_b) / ma_tuple[1]

    ma_a = 1 / (ma_a - 1)

    ma_b = 1 / (ma_b - 1)


    print('Xi={}, RatioA={:.14f}, RatioB={:.14f}'.format(ma_x, ma_a, ma_b))
```

```
print('------------------------------------------------------')
```

We now return to the field which is smaller than 1, namely to $\xi = \frac{95}{96}$. It is easy to see that if we pick $\xi_1 = \frac{193}{192} = 1 + \frac{1}{192}$ and $\xi_2 = \frac{63}{64} = 1 - \frac{1}{64}$ we get rational roots for the following anti-gravity equation ${x_1}^2 + \frac{1}{96}\xi_1 x_1 - \frac{1}{192}{\xi_1}^2 = {x_1}^3$ and gravity equation ${x_2}^2 - \frac{1}{96}\xi_2 x_2 - \frac{1}{192}{\xi_2}^2 = {x_2}^3$ for which $x_1 = \xi_1$ and $x_2 = \xi_2$, interestingly $1 + \frac{\xi_2 - \xi_1}{2} = \frac{95}{96}$ and $\frac{\xi_1 - \xi_2}{2} = \frac{1}{96}$.

Some nice relation between the roots of gravity and anti-gravity of area ratio polynomials with field strength coefficients $\xi = \frac{95}{96}$ and $\xi \cong 1.5561985371903483965638770314399$ as in (35) is considered. We saw that for $\xi \cong 1.5561985371903483965638770314399$ the following holds: $\frac{2(1-x_2)}{(x_1-1)^{\frac{1}{2}}} = 1$. There is another relation not less illuminating, $\frac{4(1-x_2)^{\frac{1}{2}}}{x_1-1}$. With low accuracy of a simple datasheet we can see that for $\xi = \frac{95}{96}$ we get $\frac{4(1-x_2)^{\frac{1}{2}}}{x_1-1} \cong 96.36912199$ and for $\xi \cong 1.5561985371903483965638770314399$ as in (35), we get $\frac{4(1-x_2)^{\frac{1}{2}}}{x_1-1} \cong 191.2741085$ which is almost 192=2*96. Multiplying these two values together we have $96.36912199 \ldots * 191.2741085 \ldots \cong 18432.9179 \approx 18432 = 2 * 96^2$, and we can see $(\frac{18432.9179}{2})^{\frac{1}{2}} \cong 96.00239033$.

Conclusion

The presented model predicts gravity not only by mass but also by electric charge. It offers a technological breakthrough by generating inertial dipoles and it offers mass ratios between particles that are not accessible through the Standard Model. (33) and (65) can only be interpreted as the existence of a fifth force of Nature with symmetry SU(4), while (24) results in a new neutrally charged particle of energy ~41.8752442118608 eV. The muon field strength coefficient is different than the electron's and Tauon field strength, which implies different physics. (33) indicates a deep relation between leptons and hadrons and especially between the Muon and the Bottom Quark.

Appendix A: Euler Lagrange minimum action equations

We assume $\sigma = 8\pi$ (from the previously discussed term, $-a_\mu a^\mu / 8\pi K$ as an energy density).

$$\mathrm{Z} = N^2 = \mathrm{P}_\mu \mathrm{P}^\mu \text{ and } U_\lambda = \frac{Z_\lambda}{Z} - \frac{Z_k P^k P_\lambda}{Z^2} \text{ and } \mathrm{L} = \frac{1}{4}\mathrm{U}^k \mathrm{U}_k$$

$$R = Ricci \text{ curvature.}$$

$$Min\ Action = Min \int_\Omega \left(R - \frac{8\pi}{\sigma} L \right) \sqrt{-g}\, d\Omega =$$

$$Min \int_\Omega \left(R - \frac{1}{4}\mathrm{U}^k \mathrm{U}_k \right) . \sqrt{-g}\, d\Omega \text{ s.t. } \sigma = 8\pi \tag{47}$$

The variation of the Ricci scalar is well known. It uses the Platini identity and Stokes theorem to calculate the variation of the Ricci curvature and reaches the Einstein tensor [30], as follows,

$\delta R = R_{\mu\nu} \delta g^{\mu\nu}$ and $\delta\sqrt{-g} = -\frac{1}{2} g_{\mu\nu} \delta g^{\mu\nu} \sqrt{-g}$ by which we infer $\delta(R\sqrt{-g}) = (R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}) \delta g^{\mu\nu}$ which will be later added to the variation of $\left((R - \frac{1}{4} U^j U_j)\right)\sqrt{-g}$ by $\delta g^{\mu\nu}$. The following Euler Lagrange equations have to hold,

$$\frac{\partial}{\partial g^{\mu\nu}} - \frac{d}{dx^m}\frac{\partial}{\partial g^{\mu\nu},_m} + \frac{d^2}{dx^m dx^s}\frac{\partial}{\partial g^{\mu\nu},_{m,s}}\left((R - \tfrac{1}{4} U^j U_j)\sqrt{-g}\right) = 0,$$

$$\frac{\partial}{\partial P} - \frac{d}{dx^m}\frac{\partial}{\partial P,_m} + \frac{d^2}{dx^m dx^s}\frac{\partial}{\partial P,_{m},_s}\left((R - \frac{1}{4} U^j U_j)\sqrt{-g}\right) = 0$$

$\mathrm{U}^k \mathrm{U}_k = \frac{Z_\mu Z^\mu}{Z^2} - \frac{(Z_s P^s)^2}{Z^3}$ which we obtain from the minimum Euler Lagrange equation because $U_\lambda P^\lambda = \frac{Z_\lambda P^\lambda}{Z} - \frac{Z_k P^k P_\lambda P^\lambda}{Z^2} = 0$. In order to calculate the minimum action Euler-Lagrange equations, we will separately treat the Lagrangians, $L = \frac{Z_\mu Z^\mu}{Z^2}$ and $L = \frac{(Z_s P^s)^2}{Z^3}$ to derive the Euler Lagrange equations of the Lagrangian $L = \frac{Z_\mu Z^\mu}{Z^2} - \frac{(Z_s P^s)^2}{Z^3} = U_\mu U^\mu$. The Euler Lagrange operator of the Ricci scalar $(\frac{\partial}{\partial g^{\mu\nu}} - \frac{d}{dx^m}\frac{\partial}{\partial (g^{\mu\nu},_m)} + \frac{d^2}{dx^m dx^s}\frac{\partial}{\partial (g^{\mu\nu},_m,_s)}$.

The reader may skip the following equations up to equation (53). Equations (53), (54) and (55) are however crucial. Note: the relation $\frac{d}{dx^\nu}\sqrt{|g|} = \Gamma^\lambda_{\lambda\nu}\sqrt{|g|}$ is used in the next equations.

$$L = \frac{(P_\lambda Z^\lambda)^2}{Z^3}\ s.t.\ Z = P_\mu P^\mu \ and\ Z_s \equiv Z,_s = \frac{dZ}{dx^s}$$

$$\frac{\partial(L\sqrt{-g}}{\partial g^{\mu\nu}}-\frac{d}{dx^m}\frac{\partial(L\sqrt{-g}\,)}{\partial g^{\mu\nu},_m}$$
$$=\left(-2\left(\frac{Z,_s P^s}{Z^3}P_\mu P_\nu P^m\right);_m+2\left(\frac{Z,_s P^s}{Z^3}\right)\left(\Gamma^{\mathrm{i}}_{\mathrm{\mu m}}P_iP_\nu P^m+\Gamma^{\mathrm{i}}_{\mathrm{\nu m}}P_\mu P_i P^m\right)\right.$$
$$+2\left(\frac{Z,_s P^s}{Z^3}\right)(P_\mu P_\nu);_m P^m-2\left(\frac{Z,_s P^s}{Z^3}\right)\left(\Gamma^{\mathrm{i}}_{\mathrm{\mu m}}P_iP_\nu P^m+\Gamma^{\mathrm{i}}_{\mathrm{\nu m}}P_\mu P_i P^m\right)$$
$$\left.+2\left(\frac{Z,_s P^s}{Z^3}\right)Z_\mu P_\nu-3\frac{(Z,_s P^s)^2}{Z^4}P_\mu P_\nu-\frac{1}{2}\frac{(Z,_s P^s)^2}{Z^3}g_{\mu\nu}\right)\sqrt{-g}=$$

$$\left(-2\left(\frac{Z,_sP^s}{Z^3}P^k\right);_k P_\mu P_\nu-2\frac{(Z,_sP^s)^2}{Z^3}\frac{P_\mu P_\nu}{Z}-\frac{(Z,_sP^s)^2}{Z^3}\frac{P_\mu P_\nu}{Z}+2\left(\frac{Z,_sP^s}{Z^3}\right)Z_\mu P_\nu-\frac{1}{2}\frac{(Z,_sP^s)^2}{Z^3}g_{\mu\nu}\right)\sqrt{-g} \quad (48)$$

$L=\frac{Z^\lambda Z_\lambda}{Z^2}\ s.t.\ Z=P_\mu P^\mu, s.t.\ Z=P_\mu P^\mu\ and\ Z_s\equiv Z,_s=\frac{dZ}{dx^s}$

$$\frac{\partial(L\sqrt{-g}}{\partial g^{\mu\nu}}-\frac{d}{dx^m}\frac{\partial(L\sqrt{-g}\,)}{\partial g^{\mu\nu},_m}=\left(-2\left(\frac{Z^mP_\mu P_\nu}{Z^2}\right);_m+2\frac{(\Gamma^{\mathrm{i}}_{\mathrm{\mu m}}\mathrm{P_iP_\nu Z^m}+\Gamma^{\mathrm{i}}_{\mathrm{\nu m}}P_iP_\mu Z^m)}{Z^2}+2\frac{(P_\mu P_\nu);_mZ^m}{Z^2}-\right.$$
$$\left.2\frac{(\Gamma^{\mathrm{i}}_{\mathrm{\mu m}}\mathrm{P_iP_\nu Z^m}+\Gamma^{\mathrm{i}}_{\mathrm{\nu m}}P_iP_\mu Z^m)}{Z^2}+\frac{Z_\mu Z_\nu}{Z^2}-2\frac{Z_sZ^s}{Z^3}P_\mu P_\nu-\frac{1}{2}\frac{Z_mZ^m}{Z^2}g_{\mu\nu}\right)\sqrt{-g}=\left(-2\left(\frac{Z^m}{Z^2}\right);_m P_\mu P_\nu-\right.$$
$$\left.2\frac{Z_sZ^s}{Z^3}P_\mu P_\nu-\frac{1}{2}\frac{Z_mZ^m}{Z^2}g_{\mu\nu}+\frac{Z_\mu Z_\nu}{Z^2}\right)\sqrt{-g} \quad (49)$$

We subtract (48) from (49)

$Z=P_\mu P^\mu, s.t.\ Z=P_\mu P^\mu\ and\ Z_s\equiv Z,_s=\frac{dZ}{dx^s}, U_\lambda=\frac{Z_\lambda}{Z}-\frac{Z_kP^kP_\lambda}{Z^2}, L=U^kU_k=\frac{Z_\lambda Z^\lambda}{Z^2}-\frac{(Z_kP^k)^2}{Z^3}$

$$\frac{\partial(L\sqrt{-g}}{\partial g^{\mu\nu}}-\frac{d}{dx^m}\frac{\partial(L\sqrt{-g}\,)}{\partial g^{\mu\nu},_m}=\left(+2\left(\frac{Z_mP^m}{Z^3}P^k\right);_k P_\mu P_\nu+2\frac{(Z_mP^m)^2}{Z^3}\frac{P_\mu P_\nu}{Z}-2\frac{Z_mP^m}{Z^3}Z_\mu P_\nu+\right.$$
$$\left.\frac{1}{2}\frac{(Z_mP^m)^2}{Z^3}g_{\mu\nu}+\frac{(Z_mP^m)^2}{Z^3}\frac{P_\mu P_\nu}{Z}+(-2\left(\frac{Z^m}{Z^2}\right);_m P_\mu P_\nu-2\frac{Z_\lambda Z^\lambda}{Z^2}\frac{P_\mu P_\nu}{Z}-\frac{1}{2}\frac{Z_\lambda Z^\lambda}{Z^2}g_{\mu\nu}+\frac{Z_\mu Z_\nu}{Z^2})\right)\sqrt{-g}=$$
$$\left(\left(+2\left(\frac{Z_mP^m}{Z^3}P^k\right);_k-2\left(\frac{Z^m}{Z^2}\right);_m\right)P_\mu P_\nu+2\frac{\left(P^\lambda Z_\lambda\right)^2}{Z^3}\frac{P_\mu P_\nu}{z}-2\frac{Z^\lambda Z_\lambda}{Z^2}\frac{P_\mu P_\nu}{Z}+\frac{1}{2}\frac{\left(P^\lambda Z_\lambda\right)^2}{Z^3}g_{\mu\nu}-\right.$$
$$\left.\frac{1}{2}\frac{Z_kZ^k}{Z^2}g_{\mu\nu}+\frac{Z_\mu Z_\nu}{Z^2}-2\left(\frac{Z_sP^s}{Z^3}\right)Z_\mu P_\nu+\frac{\left(P^\lambda Z_\lambda\right)^2}{Z^3}\frac{P_\mu P_\nu}{Z}\right)\sqrt{-g}=\left(\left(+2\left(\frac{Z_mP^m}{Z^3}P^k\right);_k-\right.\right.$$
$$\left.2\left(\frac{Z^m}{Z^2}\right);_m\right)P_\mu P_\nu+2\frac{\left(P^\lambda Z_\lambda\right)^2}{Z^3}\frac{P_\mu P_\nu}{z}-2\frac{Z^\lambda Z_\lambda}{Z^2}\frac{P_\mu P_\nu}{z}+U_\mu U_\nu-\frac{1}{2}U^\lambda U_\lambda g_{\mu\nu}\Bigg)\sqrt{-g}=$$

$$\left(U_\mu U_\nu-\frac{1}{2}U^\lambda U_\lambda g_{\mu\nu}-2U^k;_k\frac{P_\mu P_\nu}{z}\right)\sqrt{-g} \quad (50)$$

$$L=\frac{(Z^sP_s)^2}{Z^3}\quad s.t.\ Z=P^\lambda P_\lambda\ and\ Z_m=(P^\lambda P_\lambda),_m$$
$$\frac{\partial(L\sqrt{-g})}{\partial P_\mu}-\frac{d}{dx^\nu}\frac{\partial(L\sqrt{-g})}{\partial P_\mu,_\nu}=$$
$$\begin{pmatrix}-4(\frac{(Z_sP^s)}{Z^3}P^\mu P^\nu);_\nu+4\frac{(Z_sP^s)}{Z^3}\Gamma_i{}^\mu{}_\nu P^iP^\nu+\\+4\frac{(Z_sP^s)}{Z^3}P^\mu;_\nu P^\nu-4\frac{(Z_sP^s)}{Z^3}\Gamma_i{}^\mu{}_kP^iP^k+\\+2\frac{Z_mP^mZ^\mu}{Z^3}-6\frac{(Z_mP^m)^2}{Z^4}P^\mu\end{pmatrix}\sqrt{-g}=$$
$$(-4(\frac{(Z_sP^s)P^\nu}{Z^3});_\nu P^\mu+2\frac{Z_mP^mZ^\mu}{Z^3}-6\frac{(Z_mP^m)^2}{Z^4}P^\mu)\sqrt{-g}\tag{51}$$
$$L=\frac{Z^sZ_s}{Z^2}\quad s.t.\ Z=P^\lambda P_\lambda\ and\ Z_m=(P^\lambda P_\lambda),_m$$
$$\frac{\partial(L\sqrt{-g})}{\partial P_\mu}-\frac{d}{dx^\nu}\frac{\partial(L\sqrt{-g})}{\partial P_\mu,_\nu}=$$
$$\begin{pmatrix}-4(\frac{P^\mu Z^\nu}{Z^2});_\nu+\frac{4}{Z^2}\Gamma_i{}^\mu{}_kP^iZ^k+\\+\frac{4}{Z^2}P^\mu;_\nu Z^\nu-\frac{4}{Z^2}\Gamma_i{}^\mu{}_kP^iZ^k+\\-4\frac{Z_mZ^m}{Z^3}P^\mu\sqrt{-g}\end{pmatrix}\sqrt{-g}=$$
$$(-4(\frac{Z^\nu}{Z^2});_\nu-4\frac{Z_mZ^m}{Z^3})P^\mu\sqrt{-g}\tag{52}$$

We subtracted the Euler Lagrange operators of $\frac{(Z^sP_s)^2}{Z^3}\sqrt{-g}$ in (48) from the Euler Lagrange operators of $\frac{Z^\lambda Z_\lambda}{Z^2}\sqrt{-g}$ in (49) and got (50) and we will subtract (51) from (52) to get two tensor equations of gravity, these will be (53), and (55). Assuming $\sigma=8\pi$, where the metric variation equations (47), (48), (49) and (50) yield

$$\mathrm{Z}=N^2=\mathrm{P}_\mu\mathrm{P}^\mu\ ,\ U_\lambda=\frac{Z_\lambda}{Z}-\frac{Z_kP^kP_\lambda}{Z^2}\ ,\ L=\frac{1}{4}U_iU^i\ \text{ and } \mathrm{Z}=\mathrm{P^kP_k}$$
$$\frac{8\pi}{\sigma}\frac{1}{4}\begin{pmatrix}+2((\frac{(\mathrm{P}^\lambda\mathrm{P}_\lambda),_\mathrm{m}P^m}{Z^3}P^k);_k-2(\frac{Z^m}{Z^2});_m)P_\mu P_\nu+\\+2\frac{(P^\lambda Z_\lambda)^2}{Z^3}\frac{P_\mu P_\nu}{Z}-2\frac{Z^\lambda Z_\lambda}{Z^2}\frac{P_\mu P_\nu}{Z}+\\+U_\mu U_\nu-\frac{1}{2}U_\mathrm{k}U^kg_{\mu\nu}\end{pmatrix}=$$
$$\frac{8\pi}{\sigma}\frac{1}{4}(U_\mu U_\nu-\frac{1}{2}U_\mathrm{k}U^kg_{\mu\nu}-2U^k;_k\frac{P_\mu P_\nu}{Z})=R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}$$
$$\text{s.t. } R=R_{\mu\upsilon}g^{\mu\upsilon}$$
$$\text{s.t. } R_{kj}=(\Gamma_{jk}{}^P),_p-(\Gamma_{pk}{}^P),_j+\Gamma_{p\mu}{}^P\Gamma_{jk}{}^\mu-\Gamma_{pj}{}^\mu\Gamma_{k\mu}{}^P\tag{53}$$

$R_{\mu\nu}$ is the Ricci tensor and $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$ is the Einstein tensor [30]. In general, by (4) and $\sigma = 8\pi$, (53) can be written in $(-1,+1,+1,+1)$ metric convention, so $Z = |P_\mu P^\mu|$ as,

$$\frac{1}{4}(U_\mu U_\nu - \frac{1}{2}U_k U^k g_{\mu\nu} - 2U^k;_k \frac{P_\mu P_\nu}{Z}) = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} \tag{54}$$

Charge-less field: The term $-2U^k;_k \frac{P_\mu P_\nu}{Z}$ in (54) can be generalized to:

$-2((U^k;_k + U*^k;_k)/2)\frac{(P_\mu P*_\nu + P*_\mu P_\nu)/2}{Z}$ and can be zero under the following condition:

$$4(A_{\mu\nu};^\mu \frac{P*^\nu}{\sqrt{Z}} + A*_{\mu\nu};^\mu \frac{P^\nu}{\sqrt{Z}}) = U_\mu U*^\mu + U*_\mu U^\mu \Rightarrow U^k;_k + U*^k;_k = 0$$

Note: The complimentary matrix $B_{\mu\nu} = \frac{1}{\sqrt{2}}E^{\mu\nu\alpha\beta}A_{\alpha\beta}$, see few lines before (3), can be transformed to a real matrix due to the SU(2) x U(1) degrees of freedom and also be imaginary.

From (51), (52) we have, $\frac{d}{dx^\mu}(\frac{\partial}{\partial P_\mu} - \frac{d}{dx^\nu}\frac{\partial}{\partial P_{\mu},_\nu})(U_k U^k\sqrt{-g}) = W^\mu;_\mu\sqrt{-g} = 0$

We recall, $W^\mu = (\frac{\partial}{\partial P_\mu} - \frac{d}{dx^\nu}\frac{\partial}{\partial P_{\mu},_\nu})(U_k U^k\sqrt{-g})$

$$\begin{aligned}
&W^\mu = \\
&(-4(\frac{Z^\nu}{Z^2});_\nu - 4\frac{Z_m Z^m}{Z^3})P^\mu + 4(\frac{(Z_s P^s)P^\nu}{Z^3});_\nu P^\mu - 2\frac{Z_m P^m Z^\mu}{Z^3} + 6\frac{(Z_m P^m)^2}{Z^4}P^\mu = \\
&-4(\frac{Z^\nu}{Z^2});_\nu P^\mu - 4\frac{Z_m Z^m}{Z^3}P^\mu + \\
&+4(\frac{(Z_s P^s)P^\nu}{Z^3});_\nu P^\mu + 4\frac{(Z_m P^m)^2}{Z^4}P^\mu \\
&-2\frac{Z_m P^m}{Z^2}(\frac{Z^\mu}{Z} - \frac{Z_m P^m P^\mu}{Z^2}) = \\
&-4((\frac{U^k}{Z});_k + \frac{U^k U_k}{Z})P^\mu - 2\frac{Z_m P^m}{Z^2}U^\mu = 0
\end{aligned}$$

$$W^\mu;_\mu = \left(-4U^\nu;_\nu \frac{P^\mu}{Z} - 2\frac{(Z_m P^m)}{Z^2}U^\mu\right);_\mu = 0 \tag{55}$$

Appendix B: Proof of conservation

Theorem: Conservation law of the real Reeb vector.

From the vanishing of the divergence of Einstein tensor and (54), we have to prove the following in $(-1,+1,+1,+1)$ metric convention:

$$\frac{1}{4}\left(U_\mu U_\nu - \frac{1}{2}U_k U^k g_{\mu\nu} - 2U^k;_k \frac{P_\mu P_\nu}{Z}\right);^\mu = G_{\mu\nu};^\mu = (R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu});^\mu = 0 \tag{56}$$

Proof:

From the zero variation by the scalar time field (55)

$$W^\mu;_\mu = \left(-4U^\nu;_\nu \frac{P^\mu}{Z} - 2\frac{(Z_m P^m)}{Z^2}U^\mu\right);_\mu = 0 \tag{57}$$

$$-\left(2U^\nu;_\nu \frac{P^\mu}{Z}\right);_\mu = \left(\frac{(Z_m P^m)}{Z^2}U^\mu\right);_\mu \tag{58}$$

$$\left(-2U^k;_k \frac{P^\mu P^\nu}{Z}\right);_\mu = \left(\frac{(Z_m P^m)}{Z^2}U^\mu\right);_\mu P^\nu - \left(2U^k;_k \frac{P^\mu}{Z}\right)P^\nu;_\mu =$$
$$\left(\frac{(Z_m P^m)}{Z^2}U^\mu\right);_\mu P^\nu - U^k;_k \frac{Z^\nu}{Z} \tag{59}$$

Now let $t \equiv Z_m P^m$

And we have $\left(\frac{t}{Z^2}U^\mu\right);_\mu P^\nu - U^k;_k \frac{Z^\nu}{Z} = (\frac{t}{Z^2});_\mu U^\mu P^\nu + \frac{t}{Z^2}U^\mu;_\mu P^\nu - U^k;_k \frac{Z^\nu}{Z} =$

$$-U^\mu;_\mu U^\nu + (\frac{t}{Z^2});_\mu U^\mu P^\nu$$

This is because $-U^\nu = -\frac{Z^\nu}{Z} + \frac{t}{Z^2}P^\nu \Longrightarrow -U^\mu;_\mu \frac{Z^\nu}{Z} + \frac{t}{Z^2}U^\mu;_\mu P^\nu = -U^\mu;_\mu U^\nu$. Note that $-U^\nu$ is minus twice the real numbered Reeb vector. So,

$$(-2U^k;_k \frac{P^\mu P^\nu}{Z});_\mu = -U^\mu;_\mu U^\nu + (\frac{t}{Z^2});_\mu U^\mu P^\nu \tag{60}$$

Returning to the theorem we have to prove and using equation (60), we have to show,

$$\left(U^{\mu}U^{\nu}-\frac{1}{2}U_{k}U^{k}g^{\mu\nu}-2U^{k};_{k}\frac{P^{\mu}P^{\nu}}{Z}\right);^{\mu}=$$

$$U^{\mu};_{\mu}U^{\nu}+U^{\mu}U^{\nu};_{\mu}-\frac{1}{2}(U_{k};_{\mu}U_{s}+U_{k}U_{s};_{\mu})g^{ks}g^{\mu\nu}-$$

$$U^{\mu};_{\mu}U^{\nu}+(\frac{t}{Z^{2}});_{\mu}U^{\mu}P^{\nu}=$$

$$U^{\mu}U^{\nu};_{\mu}-\frac{1}{2}(U^{s}U_{s});^{\nu}+(\frac{t}{Z^{2}});_{\mu}U^{\mu}P^{\nu}=0 \tag{61}$$

Notice that

$$U^{\mu}U^{\nu};_{\mu}-\frac{1}{2}U^{s}U_{s};^{\nu}=$$

$$U^{\mu}\left((\frac{Z_{k}}{Z});_{\mu}-(\frac{t}{Z^{2}});_{\mu}P_{k}-(\frac{t}{Z^{2}})P_{k};_{\mu}\right)g^{k\nu}-$$

$$U^{s}\left((\frac{Z_{s}}{Z});_{k}-(\frac{t}{Z^{2}});_{k}P_{s}-(\frac{t}{Z^{2}})P_{s};_{k}\right)g^{k\nu}=$$

$$-U^{\mu}(\frac{t}{Z^{2}});_{\mu}P^{\nu} \tag{62}$$

Since $-(\frac{t}{Z^{2}});_{k}P_{s}U^{s}=0$ because the Reeb vector is perpendicular to the foliation kernel $\frac{P_{\lambda}}{\sqrt{Z}}$, $\frac{P^{k}}{\sqrt{Z}}\frac{U_{k}}{2}=0$.

Equation (62) is also a result of $\ln(Z)_{,k};_{\mu}U^{\mu}g^{k\nu}=\ln(Z)_{,s};_{k}U^{s}g^{k\nu}$ and of $P_{k};_{\mu}U^{\mu}g^{k\nu}=P_{s};_{k}U^{s}g^{k\nu}$.

$$U^{\mu}U^{\nu};_{\mu}-\frac{1}{2}(U^{s}U_{s});^{\nu}+(\frac{t}{Z^{2}});_{\mu}U^{\mu}P^{\nu}=-U^{\mu}(\frac{t}{Z^{2}});_{\mu}P^{\nu}+(\frac{t}{Z^{2}});_{\mu}U^{\mu}P^{\nu}=0 \tag{63}$$

and we are done.

Appendix C: Generalization to more than one Reeb vector

Given the previous fields $\frac{\mathrm{P_k}}{\sqrt{\mathrm{Z}}}$ and $\frac{U_\mu}{2}$ and additional Reeb vector fields $\frac{U(2)_\mu}{2}$, $\frac{S_\mu}{2}$, $\frac{W_\mu}{2}$, $\frac{T_\mu}{2}$,

The following Lagrangian can be defined with the determinant of the metric g:

$$L = \begin{vmatrix} 1 & 0 \\ 0 & \frac{U^k U_k^* + U^{*k} U_k}{8} \end{vmatrix} \sqrt{-\mathrm{g}} +$$

$$\begin{vmatrix} 1 & 0 & \frac{\mathrm{P_k} U(2)^{*k} + \mathrm{P^*}_\mathrm{k} U(2)^{*k}}{2\sqrt{2\mathrm{Z}}} \\ 0 & \frac{U^k U_k^* + U^{*k} U_k}{8} & \frac{U(2)^k U_k^* + U(2)^{*k} U_k}{8} \\ \frac{\mathrm{P_k} U(2)^{*k} + \mathrm{P^*}_\mathrm{k} U(2)^{*k}}{2\sqrt{2\mathrm{Z}}} & \frac{U(2)^k U_k^* + U(2)^{*k} U_k}{8} & \frac{U(2)^k U(2)_k^* + U(2)^{*k} U(2)_k}{8} \end{vmatrix}^{\frac{1}{2}} \sqrt{-\mathrm{g}} +$$

$$\begin{vmatrix} 1 & \frac{p_\mu S^{*\mu} + p_\mu^* S^\mu}{2\sqrt{2\mathrm{Z}}} & \frac{p_\mu W^{*\mu} + p_\mu^* W^\mu}{2\sqrt{2\mathrm{Z}}} & \frac{p_\mu T^{*\mu} + p_\mu^* T^\mu}{2\sqrt{2\mathrm{Z}}} \\ \frac{p_\mu S^{*\mu} + p_\mu^* S^\mu}{2\sqrt{2\mathrm{Z}}} & \frac{S_\mu S^{*\mu} + S_\mu^* S^\mu}{8} & \frac{S_\mu W^{*\mu} + S_\mu^* W^\mu}{8} & \frac{S_\mu T^{*\mu} + S_\mu^* T^\mu}{8} \\ \frac{p_\mu W^{*\mu} + p_\mu^* W^\mu}{2\sqrt{2\mathrm{Z}}} & \frac{W_\mu S^{*\mu} + W_\mu^* S^\mu}{8} & \frac{W_\mu W^{*\mu} + W_\mu^* W^\mu}{8} & \frac{W_\mu T^{*\mu} + W_\mu^* T^\mu}{8} \\ \frac{p_\mu T^{*\mu} + p_\mu^* T^\mu}{2\sqrt{2\mathrm{Z}}} & \frac{T_\mu S^{*\mu} + T_\mu^* S^\mu}{8} & \frac{T_\mu W^{*\mu} + T_\mu^* W^\mu}{8} & \frac{T_\mu T^{*\mu} + T_\mu^* T^\mu}{8} \end{vmatrix}^{\frac{1}{3}} \sqrt{-\mathrm{g}} \qquad (64)$$

Possibly a fifth force of Nature is described by the following SU(4) symmetry Lagrangian of 4 Reeb vectors: $\frac{\aleph_\mu}{2}, \frac{\beth_\mu}{2}, \frac{\gimel_\mu}{2}, \frac{\daleth_\mu}{2}$, with Hebrew letters Alef, Beit, Gimmel, Dalet,

$$\begin{vmatrix} \frac{\aleph_\mu \aleph^{*\mu} + \aleph_\mu^* \aleph^\mu}{8} & \frac{\aleph_\mu \beth^{*\mu} + \aleph_\mu^* \beth^\mu}{8} & \frac{\aleph_\mu \gimel^{*\mu} + \aleph_\mu^* \gimel^\mu}{8} & \frac{\aleph_\mu \daleth^{*\mu} + \aleph_\mu^* \daleth^\mu}{8} \\ \frac{\aleph_\mu \beth^{*\mu} + \aleph_\mu^* \beth^\mu}{8} & \frac{\beth_\mu \beth^{*\mu} + \beth_\mu^* \beth^\mu}{8} & \frac{\beth_\mu \gimel^{*\mu} + \beth_\mu^* \gimel^\mu}{8} & \frac{\beth_\mu \daleth^{*\mu} + \beth_\mu^* \daleth^\mu}{8} \\ \frac{\aleph_\mu \gimel^{*\mu} + \aleph_\mu^* \gimel^\mu}{8} & \frac{\gimel_\mu \beth^{*\mu} + \gimel_\mu^* \beth^\mu}{8} & \frac{\gimel_\mu \gimel^{*\mu} + \gimel_\mu^* \gimel^\mu}{8} & \frac{\gimel_\mu \daleth^{*\mu} + \gimel_\mu^* \daleth^\mu}{8} \\ \frac{\aleph_\mu \daleth^{*\mu} + \aleph_\mu^* \daleth^\mu}{8} & \frac{\daleth_\mu \beth^{*\mu} + \daleth_\mu^* \beth^\mu}{8} & \frac{\daleth_\mu \gimel^{*\mu} + \daleth_\mu^* \gimel^\mu}{8} & \frac{\daleth_\mu \daleth^{*\mu} + \daleth_\mu^* \daleth^\mu}{8} \end{vmatrix}^{\frac{1}{4}} \sqrt{-\mathrm{g}} \qquad (65)$$

The determinant of two Reeb vectors can help to understand the roots in (30), (31), (32), and (33). It describes accelerations in two perpendicular planes. Three Reeb vectors describe accelerations in the foliation perpendicular to P_μ.

Appendix D: Another way to derive the Reeb vector

We may now write the Lie derivative [31] of $\frac{P_i}{\sqrt{Z}}$ with respect to the vector field $\frac{P^{*m}}{\sqrt{Z}}$,

$$Lie\left(\frac{P^{*m}}{\sqrt{Z}},\frac{P_i}{\sqrt{Z}}\right)=\frac{P^{*m}}{\sqrt{Z}}\left(\frac{P_i}{\sqrt{Z}}\right),_m+\left(\frac{P^{*m}}{\sqrt{Z}}\right),_i\frac{P_m}{\sqrt{Z}} \tag{66}$$

In which the second term is positive because the differentiated $\frac{P_i}{\sqrt{Z}}$ vector has a low index.
The first term becomes,

$$\frac{P^{*m}}{\sqrt{Z}}\left(\frac{P_i}{\sqrt{Z}}\right),_m=\frac{P^{*m}P_{i,m}}{Z}-\frac{P^{*m}}{\sqrt{Z}}\frac{P_iZ_m}{2Z^{3/2}}=\frac{P^{*m}P_{i,m}}{Z}-\frac{P^{*m}Z_mP_i}{2Z^2} \tag{67}$$

The second term is,

$$\left(\frac{P^{*m}}{\sqrt{Z}}\right),_i\frac{P_m}{\sqrt{Z}}=\frac{P^{*m},_iP_m}{Z}-\frac{P^{*m}P_mZ_i}{2Z^2}=\frac{P^{*m},_iP_m}{Z}-\frac{Z_i}{2Z} \tag{68}$$

We add (67) and (68) to get (66) and notice that $\frac{P^{*m}P_{i,m}}{Z}+\frac{P^{*m},_iP_m}{Z}=\frac{P^{*m}P_{m,i}}{Z}+\frac{P^{*m},_iP_m}{Z}=\frac{Z_i}{Z}$ from which (66) becomes

$$Lie\left(\frac{P^{*m}}{\sqrt{Z}},\frac{P_i}{\sqrt{Z}}\right)=\frac{Z_i}{Z}-\frac{Z_i}{2Z}-\frac{P^{*m}Z_mP_i}{2Z^2}=\frac{Z_i}{2Z}-\frac{P^{*m}Z_mP_i}{2Z^2}=\frac{U_i}{2} \tag{69}$$

Appendix E: 95/96, the precursor of the inverse Fine Structure Constant and of the muon/electron mass ratio

Results (24), (36), (40), (41), (42), were not reached immediately. There was one finding that was a total serendipity that later lead to these results. The observation was the following, given a scaling factor 1+d of area addition with d=1 as a maximal value, 1+d = 2.

$$(1+\alpha)^{95}<2\ \wedge(1+\alpha)^{96}>2 \tag{70}$$

More precisely

$$\aleph=\left(2^{\frac{1}{96}}-1\right)^{-1}\cong 137.999325615 \tag{71}$$

And

$$\beth=\left(2^{\frac{1}{95}}-1\right)^{-1}\cong 136.5566369 \tag{72}$$

And the geometric average is:

$$\sqrt{\aleph\beth}\cong 137.27608605 \tag{73}$$

Which is close to the result from (40), 137.0359990368270076.

An immediate observation is

$$א = \left(\frac{2-2^{\frac{95}{96}}}{2^{\frac{95}{96}}}\right)^{-1} \tag{74}$$

And

$$ב = \left(\frac{2^{\frac{96}{95}}-2}{2}\right)^{-1} \tag{75}$$

Where we expressed a power which is close to 1, namely $\xi = \frac{95}{96}$ and $\xi^{-1} = \frac{96}{95}$. as such, ξ was nominated as polynomial coefficient because it was in the range between 0 and 2, unlike $\xi = \frac{4}{\pi}$ which has a geometric interpretation thanks to Ettore Majorana, $\xi = \frac{95}{96}$ seems to have an algebraic meaning.

We continue with a rather surprising relation

$$\left(2^{\frac{1}{95*96}} - 1\right)^{-1} \cong 13{,}156.87877924 \tag{76}$$

And it is quite easy to notice the following:

$$\frac{1}{96(1+96^{-2})}\left(2^{\frac{1}{95*96}} - 1\right)^{-1} \cong 137.03595126474 \tag{77}$$

which is very close to the inverse Fine Structure Constant. Actually, if we replace the factor $\frac{1}{96(1+96^{-2})}$ by $\frac{1}{n(1+n^{-2})}$ for some integer n, the closest result to the inverse Fine Structure Constant is when n=96

In fact

$$\frac{\left(2^{\frac{1}{95*96}}-1\right)^{-1}}{137.0359990368270076} \cong 96.010383196499723 \cong 96(1 + 96.1546032^{-2}) \tag{78}$$

See (40). The factor $\frac{1}{95*96}$ can be seen as

$$\frac{1}{95*96} = \frac{95}{96} + \frac{96}{95} - 2 \tag{79}$$

The factor $95 * 96$ found expression in (41), (42) and is the final missing piece in the puzzle. It is the bridge between trigonometry and electro-gravitational polynomials (35) which resulted in: $\xi \cong 1.556198537190348396563877031439915299415588378906$ and $\frac{1}{2}(1 - g_2)^{-4} \cong$

$607276.536800682428292930126$2, provided here with more accuracy if required for further research.

In (78) plugging in $\frac{4}{\pi}$ from (24) instead of 2 and dividing by $2 * 137.0359990368270076^2$ instead of by 137.0359990368270076 we get another indication of a deep theoretical relation,

$$\frac{\left(\left(\frac{4}{\pi}\right)^{\frac{1}{95*96}} - 1\right)^{-1}}{2*137.0359990368270076^2} \cong 1 + (2 * 95.974269533437)^{-1} \quad (80)$$

We now explore another approach, exponential perturbation of the field strength coefficient $\frac{95}{96}$.

This approach was not further investigated due to numerical stability issues, but the author finds it quite interesting. The field strength coefficient $\frac{95}{96}$ that appears in (23) is the lowest among 3 coefficients $\frac{95}{96}, \frac{4}{\pi}, 1.5561985371903484$ ... . At first this fact was an incentive to search for a relation between the fine structure constant and perturbations around the value $\frac{95}{96}$.

We return to (23):

$$\frac{192a^2 + 2\frac{95}{96}a - (\frac{95}{96})^2}{192} = a^3 \; and \; \frac{192b^2 - 2\frac{95}{96}b - (\frac{95}{96})^2}{192} = b^3 \quad (81)$$

And to the multiplication in (23) $\frac{1}{(a-1)(1-b)} \cong \; 12202.88874066467724$.

We look at the following exponential $\frac{n-1}{n}$ perturbation of the coefficient $\frac{95}{96}$,

$$\frac{192c^2 + 2\left(\frac{95}{96}\right)^{\frac{n-1}{n}} c - (\frac{95}{96})^{2\frac{n-1}{n}}}{192} = c^3 \; and \; \frac{192d^2 - 2\left(\frac{95}{96}\right)^{\frac{n-1}{n}} d - (\frac{95}{96})^{2\frac{n-1}{n}}}{192} = d^3 \quad (82)$$

And we check how relatively close is $(c-1)(1-d)$ to $(a-1)(1-b)$.

The calculation is:

$$Relative\ error = \frac{(c-1)(1-d)}{(a-1)(1-b)} - 1 \quad (83)$$

The strange fact is that

$\alpha^{-1} = \frac{2}{n}\left(\frac{(c-1)(1-d)}{(a-1)(1-b)} - 1\right)$ approximates the inverse fine structure constant. Not as good as (40), (41), (42) but good enough to trigger interest. The last term can be written as in (40) $\alpha^{-1} = \frac{2}{\cos(\eta)}$ for $\eta \equiv cos^{-1}(2\alpha)$. It turns out that α^{-1} is maximal or locally maximal at $n = 96^4 - 805$ or if n is allowed to take real values,

$$n \cong 96^4 - 805.9334 \tag{84}$$

$$\alpha^{-1} \cong 137.0158482935 \tag{85}$$

Putting the terms together:

$$\frac{192a^2+2\frac{95}{96}a-(\frac{95}{96})^2}{192} = a^3 \ and \ \frac{192b^2-2\frac{95}{96}b-(\frac{95}{96})^2}{192} = b^3 \tag{86}$$

$$\frac{192c^2 + 2\left(\frac{95}{96}\right)^{\frac{n-1}{n}} c - (\frac{95}{96})^{2\frac{n-1}{n}}}{192} = c^3 \ and \ \frac{192d^2 - 2\left(\frac{95}{96}\right)^{\frac{n-1}{n}} d - (\frac{95}{96})^{2\frac{n-1}{n}}}{192} = d^3$$

$$\max_n \frac{2}{n}\left(\frac{(c-1)(1-d)}{(a-1)(1-b)} - 1\right) \cong 137.015848292861875279413652606308460235595703,$$
$$n \cong 96^4 - 805.933$$

See appendix G for the code in Python for (81)-(86). Consider the same type of perturbation of the field strength $\xi = \frac{4}{\pi}$,

$$\frac{192a^2+2\frac{4}{\pi}a-(\frac{4}{\pi})^2}{192} = a^3 \ and \ \frac{192b^2-2\frac{4}{\pi}b-(\frac{4}{\pi})^2}{192} = b^3 \tag{86.1}$$

$$\frac{192c^2 + 2\left(\frac{4}{\pi}\right)^{\frac{n-1}{n}} c - (\frac{4}{\pi})^{2\frac{n-1}{n}}}{192} = c^3 \ and \ \frac{192d^2 - 2\left(\frac{4}{\pi}\right)^{\frac{n-1}{n}} d - (\frac{4}{\pi})^{2\frac{n-1}{n}}}{192} = d^3$$

$$\max_n \frac{2}{n}\left(\frac{(c-1)(1-d)}{(a-1)(1-b)} - 1\right) \cong 136.4^{\frac{1}{2}}$$

Which is close to the square root of the inverse Fine Structure Constant with $n \cong 96^4 - 140631.4697265625$. In both cases, numerical stability issues in (86) and (86.1) made it very difficult to check how close such exponential perturbations of the field strength coefficient can be to the inverse Fine Structure Constant trough the error in the polynomial roots. Numerical stability does exist up to $n = 96^3$. Before we proceed, consider the following, $\xi = (\frac{4}{\pi})^{1+\frac{1}{151.06357822765725984}}$ which is approximately $\frac{4}{\pi}(1+\frac{1}{624.85524})$, then it is easy to check that

$$\frac{192a^2 + 2\xi a - \xi^2}{192} = a^3 \ and \ \frac{192b^2 - 2\xi b - \xi^2}{192} = b^3$$

$$\frac{192c^2 + 2\frac{2}{\xi}c - \left(\frac{2}{\xi}\right)^2}{192} = c^3 \ and \ \frac{192d^2 - 2\frac{2}{\xi}d - \left(\frac{2}{\xi}\right)^2 \frac{4}{\pi})^{2\frac{n-1}{n}}}{192} = d^3 \Rightarrow$$

$$\frac{(c-1)(1-d)}{(a-1)(1-b)} \cong 1 \tag{86.2}$$

This result is expected from $\xi = 2^{\frac{1}{2}} = \frac{2}{\xi}$ but not from a field strength so close to $\frac{4}{\pi}$. It is easy to see that from $\xi = 1.25$ to $\xi = 1.5$, (86.2) is very close to 1 within %1 but not as close as when $\xi = (\frac{4}{\pi})^{1+\frac{1}{151.06357822765725984}}$ or when trivially $\xi = 2^{\frac{1}{2}} = \frac{2}{\xi}$.

The Fine Structure Constant as a result of Poisson Distribution of events within radius r:

We proceed with the methods we have discussed until now. Consider the following expression,

$$f(x) = xe^{-x} \tag{87}$$

which is the Poison distribution for one event and with $\lambda = x$.

Consider the following perturbation equations in two variables in x around 1.

$$\eta = f\left(1 - \frac{1}{a}\right) = f\left(1 + \frac{1}{b}\right) \tag{88}$$

With the following condition for a wide range of $\eta > 10000$,

$$\alpha^{-2} = (-\ln(\eta) - 1)^{-1} \text{ and } 2\left(\frac{1}{b} + \frac{1}{a}\right)^{-1} \cong \alpha^{-1}2^{-\frac{1}{2}} \tag{89}$$

Then the system of equations (88), (89) approximates the Fine Structure Constant with the following approximated solution:

$$a \cong 97.2332790992 \tag{90}$$

$$b \cong 96.56660927693$$

$$\alpha^{-2} = (-\ln(\eta) - 1)^{-1} \cong 18778.86503$$

$$2\left(\frac{1}{b} + \frac{1}{a}\right)^{-1} = \alpha^{-1}2^{-\frac{1}{2}} \cong 96.89879752$$

$$\text{With } \alpha^{-1} \approx 137.03559363$$

These estimates can be greatly improved with better numerical precision than that of an Excel datasheet, however, this paper does not deal with the Causal Set interpretation of the presented theory and chooses to focus on other subjects. Also, (90) depends on the choice of η.

The Causal Set interpretations can be written as Probability(n=k) = $\frac{\xi^k e^{-\xi}}{k!}$ where k is the number of events within a sphere of some small radius r and n is the number of events if this number has the Poisson distribution.

Appendix F: The Python code for (40) and for the remark after (40) and its output

```
import numpy as NP


def function_cubic_viete(a, b, c, d): # If all roots are real.

    # Viete's formula when all roots are real.

    b2 = NP.longdouble(b * b)
    b3 = NP.longdouble(b2 * b)
    a2 = NP.longdouble(a * a)
    a3 = a2 * a

    p = (3 * a * c - b2) / (3 * a2)

    q = (2 * b3 - 9 * a * b * c + 27 * a2 * d) / (27 * a3)

    offset = b / (3 * a)
    t1 = 2 * NP.sqrt(-p / 3) * NP.cos(NP.arccos(NP.sqrt(-3 / p) \
                                        * (3 * q) / (2 * p)) / 3)
    t2 = 2 * NP.sqrt(-p / 3) * NP.cos(NP.arccos(NP.sqrt(-3 / p) * \
                                        (3 * q) / (2 * p)) / 3 -
                                NP.pi / 3)
    t3 = 2 * NP.sqrt(-p / 3) * NP.cos(NP.arccos(NP.sqrt(-3 / p) * \
                                        (3 * q) / (2 * p)) / 3 -
                                2 * NP.pi / 3)
    x1 = t1 - offset
    x2 = t2 - offset
    x3 = t3 - offset
```

```
    return (x1, x2, x3)


def function_fsc_polynomials(): # If all roots are real.

    fp_f, fp_a, fp_b = 1, 1, 1
    fp_start,  fp_end = 1.556, NP.pi / 2

    for i in range(2000):
        # Get the biggest roots. These are the closest to 1.
        # One is above 1 and one is below 1.

        fp_f = (fp_start + fp_end) * 0.5

        fp_a, _, _ = function_cubic_viete(1, -1, -fp_f / 96,
                                          (fp_f * fp_f) / 192)

        fp_b, _, _ = function_cubic_viete(1, -1, fp_f / 96,
                                          (fp_f * fp_f) / 192)

        fp_result_middle = 1/NP.sqrt(fp_a-1) - 0.5/(1-fp_b)

        if fp_result_middle >= 0:
            fp_end = fp_f
        else:
            fp_start = fp_f

    fp_s = 1/(1 - fp_b)
    fp_s *= fp_s
```

```
fp_s *= fp_s * 0.5
fp_xi = fp_f

print('1/(x1-1): %.42lf\n1/(1-x2): %.42lf' %(1/(fp_a-1), 1/(1-fp_b)))
print('Xi: %.42lf\ns=0.5/(1-x2)^4: %.42lf' %(fp_f, fp_s))

fp_f = 4 / NP.pi
# Get the biggest roots. These are the closest to 1.
# One is above 1 and one is below 1.
fp_a, _, _ = function_cubic_viete(1, -1, -fp_f / 96, (fp_f * fp_f) / 192)
fp_b, _, _ = function_cubic_viete(1, -1, fp_f / 96, (fp_f * fp_f) / 192)
fp_mul = (fp_a - 1) * (1 - fp_b)

fp_inv_fsc = 2 / NP.cos( fp_xi * (1 + 1/NP.power(fp_s,1/(1+fp_mul))))

print('Inv FSC: %.42lf' %(fp_inv_fsc))

fp_p2 = fp_mul
fp_start,  fp_end = fp_mul, fp_mul + 0.00001

for i in range(2000):
    # Get the biggest roots. These are the closest to 1.
    # One is above 1 and one is below 1.

    fp_f = (fp_start + fp_end) * 0.5

    fp_result_middle = \
        fp_s * (2 - 1/(96*96*fp_f)) - NP.power(fp_s, 1/(1+fp_f))
```

```
        if fp_result_middle >= 0:
            fp_end = fp_f
        else:
            fp_start = fp_f

    fp_p = 1/NP.sqrt(fp_mul)
    fp_miracle_p = 1/NP.sqrt(fp_f)
    fp_relative_p_error = fp_p / (fp_p - fp_miracle_p)

    print('P: %.42lf\nMiracle P: %.48lf\nRelative error in P: %.48lf^-1'
          % (fp_p, fp_miracle_p, fp_relative_p_error))

function_fsc_polynomials()

'''
Output when run from PyCharm and Python 3.6:
1/(x1-1): 275.516908918643935066938865929841995239257812
1/(1-x2): 33.197404050235356010034593055024743080139160
Xi: 1.556198537190348396563877031439915299415588
s=0.5/(1-x2)^4: 607276.536800682428292930126190185546875000000000
Inv FSC: 137.035999036827007557803881354629993438720703
P: 96.069177214886295246287772897630929946899414
Miracle P: 96.069175812725177365791751071810722351074218750000
Relative error in P:
68515077.183215767145156860351562500000000000000000000000^-1
'''
```

Appendix G: The Python code for (81)-(86)

```
import numpy as NP


def function_cubic_viete(a, b, c, d): # If all roots are real.

    # Viete's formula when all roots are real.

    b2 = NP.longdouble(b * b)
    b3 = NP.longdouble(b2 * b)
    a2 = NP.longdouble(a * a)
    a3 = a2 * a

    p = (3 * a * c - b2) / (3 * a2)

    q = (2 * b3 - 9 * a * b * c + 27 * a2 * d) / (27 * a3)

    offset = b / (3 * a)
    t1 = 2 * NP.sqrt(-p / 3) * NP.cos(NP.arccos(NP.sqrt(-3 / p) \
                                        * (3 * q) / (2 * p)) / 3)
    t2 = 2 * NP.sqrt(-p / 3) * NP.cos(NP.arccos(NP.sqrt(-3 / p) * \
                                        (3 * q) / (2 * p)) / 3 -
                               NP.pi / 3)
    t3 = 2 * NP.sqrt(-p / 3) * NP.cos(NP.arccos(NP.sqrt(-3 / p) * \
                                        (3 * q) / (2 * p)) / 3 -
                               2 * NP.pi / 3)
    x1 = t1 - offset
    x2 = t2 - offset
    x3 = t3 - offset
```

```
    return (x1, x2, x3)

def function_f_polynomials(fp_n=96*96*96*96): # If all roots are real.

    fp_f = 95/96

    fp_a, _, _ = function_cubic_viete(1, -1, -fp_f / 96,
                                  (fp_f * fp_f) / 192)

    fp_b, _, _ = function_cubic_viete(1, -1, fp_f / 96,
                                  (fp_f * fp_f) / 192)

    fp_mul1 = (fp_a - 1)*(1 - fp_b)

    fp_f = NP.power(fp_f, (fp_n-1)/fp_n)

    fp_a, _, _ = function_cubic_viete(1, -1, -fp_f / 96,
                                  (fp_f * fp_f) / 192)

    fp_b, _, _ = function_cubic_viete(1, -1, fp_f / 96,
                                      (fp_f * fp_f) / 192)

    fp_mul2 = (fp_a - 1)*(1 - fp_b)

    fp_combine = 2/(fp_n *(fp_mul2/fp_mul1-1))

    #print('%.42lf' %fp_combine)
```

```
    return fp_combine


def main():
    ma_best_val = 0
    ma_best_m = 0

    #function_f_polynomials(96 * 96 * 96 * 96 - 1)
    #function_f_polynomials(96 * 96 * 96 * 96)
    #function_f_polynomials(96 * 96 * 96 * 96 + 1)

    print('Coarse search:')
    for i in range(-1000, 1000):
        ma_r =  function_f_polynomials(96 * 96 * 96 * 96 - i)
        if ma_best_val < ma_r:
            ma_best_val = ma_r
            ma_best_m = i

    print('Best value %.42lf' %ma_best_val)
    print('Best m = 96^4-%d' % ma_best_m)

    print('Fine search:')
    ma_best_val = 0.0
    ma_best_m = 0.0

    for i in range(8050000-10000, 8050000+10000):
        ma_d = i/10000
        ma_r = function_f_polynomials(96 * 96 * 96 * 96 - ma_d)
```

```
            if ma_best_val < ma_r:
             Fappen   ma_best_val = ma_r
                ma_best_m = ma_d


    print('Best value %.42lf' %ma_best_val)
    print('Best m = 96^4-%.42lf' % ma_best_m)


    '''
    Coarse search:
    Best value 137.015846787740116496934206224977970123291016
    Best m = 96^4-805
    Fine search:
    Best value 137.015848292861875279413652606308460235595703
    Best m = 96^4-805.932999999999927240423858165740966796875 00
    '''
if __name__ == '__main__':
    main()
```

Appendix H – Causality conservation theorem

Theorem: If p is real, any monotone function $f(p)$, called causality function will yield the same Reeb vector. The reader is advised to check the case when p is an imaginary function. Then the Reeb vector is defined as $\frac{u_\nu}{2} = \frac{z_\nu}{2z} - \frac{z_k}{2z^2} p *^k p_\nu$.

Proof:

We will use capital letters for $P = f(p)$ and as in previous pages, $z = p_\lambda p^\lambda$ and here $Z = P_\lambda P^\lambda$.

$$P = f(p)$$

$$P_\mu = f'(p) p_\mu$$

$$Z = f'(p) p_\mu f'(p) p^\mu = f'(p)^2 z$$

$$\frac{Z_\nu}{Z} = \frac{2f'(p)f''(p)p_\nu z}{f'(p)^2 z} + \frac{f'(p)^2 z_\nu}{f'(p)^2 z} = \frac{2f''(p)p_\nu}{f'(p)} + \frac{z_\nu}{z}$$

$$U_\nu = \frac{2f''(p)p_\nu}{f'(p)} + \frac{z_\nu}{z} - (\frac{2f''(p)p_k}{f'(p)} + \frac{z_k}{z})\frac{f'(p)p^k f'(p)p_\nu}{f'(p)^2 z}$$

$$U_\nu = \frac{2f''(p)p_\nu}{f'(p)} - \frac{2f''(p)p_\nu}{f'(p)} + \frac{z_\nu}{z} - \frac{z_k}{z^2}p^k p_\nu = \frac{z_\nu}{z} - \frac{z_k}{z^2}p^k p_\nu = u_\nu$$

$$\frac{U_\nu}{2} = \frac{u_\nu}{2} \tag{91}$$

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